Combining Discrimination Diagnostics to Identify Sources of Statistical Discrimination

Online Appendix

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A Example: statistical differences may affect bias in predictions

To simplify notation, we omit the conditioning on (G_i, Z_i) and the subscript i, so (3) can be written as $R = 1\{p_j \leq t_j\}$. Assume that judges have a normally distributed prior of the true conditional expectation, $\hat{p}_j \sim \mathcal{N}(\mu_j^p, \sigma_{pj}^2)$, where σ_{pj}^2 measures how confident the judge is about the prior. Judges observe a noisy signal $\widetilde{p}_j \sim \mathcal{N}(p, \sigma_{sj}^2)$ that is centered around the true conditional expectation, p, with the variance σ_{sj}^2 accounting for the quality of the signal or the ability of the judge to extract information from it. It follows that the posterior probability, p_j, is given by

$$p_j = \gamma_j \widetilde{p}_j + (1-\gamma_j)\hat{p}_j, \tag{A.1}$$

with $\gamma_j = \frac{\sigma_{pj}^2}{\sigma_{pj}^2 + \sigma_{sj}^2}$ being the signal to noise ratio. Intuitively, if σ_{sj}^2 is large relative to σ_{pj}^2, the signal does not give too much information and, therefore, the judge puts more weight on the prior to compute the prediction.

Using the fact that $p_j = p + b_j$, we have

$$\gamma_j \widetilde{p}_j + (1-\gamma_j)\hat{p}_j = p + b_j. \tag{A.2}$$

If we define $b_j^p = \mu_j^p - p$ as the structural bias in the prior, then replacing in the expectation of (A.2) yields

$$\mathbb{E}[b_j] = (1-\gamma_j) b_j^p. \tag{A.3}$$

That is, the expected bias of a judge is a fraction of the structural bias in the prior. Then, bias is increasing in the structural bias in the prior, but it is also increasing in the signal noise relative to the prior noise because $1-\gamma_j = \frac{\sigma_{sj}^2}{\sigma_{pj}^2 + \sigma_{sj}^2}$. This expression implies that even if the structural bias in the prior does not vary with G_i, it is still the case that statistical differences may lead to different γ_j by G_i and, therefore, to prejudice.

B Derivation of equation (13)

Using

$$R(G_i, Z_i, j(i)) \equiv \widetilde{R}(G_i, Z_i, j(i), \epsilon_i, Y_i^*) = 1\left\{\epsilon_i \leq h_{j(i)}(G_i, Z_i) - Y_i^*\right\}, \tag{B.1}$$

and an iterated expectations argument, we have

$$\begin{aligned} \mathbb{E}\left[R(G_i, Z_i, j(i))|G_i\right] &= \mathbb{E}\left[\mathbb{E}\left[R(G_i, Z_i, j(i))|G_i, Y_i^*\right]|G_i\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[\widetilde{R}(G_i, Z_i, j(i), \epsilon_i, Y_i^*)|G_i, Y_i^*\right]|G_i\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}\left[\widetilde{R}(G_i, Z_i, j(i), \epsilon_i, Y_i^*)|G_i, Z_i, j(i), Y_i^*\right]|G_i, Y_i^*\right]|G_i\right], \end{aligned} \tag{B.2}$$

so we can proceed as in the main text by solving the inner expectations first. Using A1 and A2, we have that:

$$\mathbb{E}\left[\widetilde{R}(G_i, Z_i, j(i), \epsilon_i, Y_i^*)|G_i, Z_i, j(i), Y_i^*\right] = \frac{h_{j(i)}(G_i) - Y_i^* - C(G_i, j(i), Y_i^*)}{\sigma(G_i, j(i), Y_i^*)}, \tag{B.3}$$

so

$$\mathbb{E}\left[R(G_i, Z_i, j(i))|G_i, Y_i^*\right] = \mathbb{E}\left[\frac{h_{j(i)}(G_i) - Y_i^* - C(G_i, j(i), Y_i^*)}{\sigma(G_i, j(i), Y_i^*)}|G_i, Y_i^*\right]. \tag{B.4}$$

Under A3, we can write

$$\mathbb{E}\left[\mathbb{E}\left[R(G_i, Z_i, j(i))|G_i, Y_i^*\right]|G_i\right] = \frac{1}{\sigma_g}\left(\mathbb{E}\left[h_{j(i)}(g)\right] - \mathbb{E}\left[Y_i^*|G_i = g\right]\right), \tag{B.5}$$

where we use the fact that $\mathbb{E}[\epsilon_i|G_i, Z_i] = 0$, so $\mathbb{E}[\epsilon_i|G_i] = 0$ using an argument of iterated expectations and, therefore, $\mathbb{E}\left[C(G_i, j(i), Y_i^*)|G_i\right] = 0$. Then

$$\begin{aligned} \mathbf{R} &= \frac{1}{\sigma_1}\left(\mathbb{E}\left[h_{j(i)}(1)\right] - \mathbb{E}\left[Y_i^*|G_i = 1\right]\right) - \frac{1}{\sigma_0}\left(\mathbb{E}\left[h_{j(i)}(0)\right] - \mathbb{E}\left[Y_i^*|G_i = 0\right]\right) \\ &= \frac{\mathbf{P}}{\sigma_1} + \frac{(\sigma_0 - \sigma_1)\mathbb{E}\left[h_{j(i)}(0)\right]}{\sigma_0\sigma_1} - \frac{\mathbb{E}[Y_i^*|G_i = 1]}{\sigma_1} + \frac{\mathbb{E}[Y_i^*|G_i = 0]}{\sigma_0}. \end{aligned} \tag{B.6}$$