Equality of Opportunity vs Equality of Outcomes: A Theory of Optimal Predistribution*

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Abstract

We generalize the Mirrleesian framework to incorporate two features central to public policy debates: (i) the use of "predistributive" policies, such as public education, that reduce inequality before individuals enter the labor market and (ii) social preferences for equality of opportunity. In our model, income inequality arises from heterogeneity in two dimensions: preferences for work and family background. The government accepts inequality arising from preferences, but seeks to offset inequality arising from family background. Taxes and transfers redistribute income regardless of the source of income differences, whereas predistributive policies affect skills based solely on family background. We derive simple policy rules in terms of estimable parameters, providing a normative framework for interpreting the empirical literature on economic mobility. Governments optimally place greater weight on predistribution relative to redistribution when the earnings elasticity with respect to taxes is large and when the earnings elasticity with respect to education is large. Predistribution is also favored when income inequality is driven primarily by preference heterogeneity, because predistribution targets background disadvantage while preserving preference-driven differences. Calibrations based on existing empirical estimates suggest that, relative to the Mirrleesian benchmark, optimal policy is more progressive overall, but the tax and transfer system is less progressive.

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1 Introduction

The canonical normative framework in public economics asks how a social planner should use taxes and transfers to redistribute income in a world with heterogeneous skill (Mirrlees 1971). This framework has generated important insights into the design of tax policy and provided tractable formulas for optimal tax rates based on estimable parameters (Diamond 1998; Saez 2001). Optimal income taxes are more progressive when social preferences for income equality are stronger and when labor supply is less responsive to taxation.

Although the Mirrleesian framework is a valuable benchmark, it abstracts from two central features of debates about government policies. First, in the Mirrleesian framework, the planner redistributes incomes solely through taxes and transfers. In practice, governments devote substantial resources to reducing pre-market inequality—often termed *predistribution*—through policies ranging from education provision to healthcare and nutritional assistance for children to place-based investments. In the United States, government expenditures on education alone exceeded $1.4 trillion in 2025, or about 4.7% of GDP (U.S. Bureau of Economic Analysis 2026). Western European countries spend similar amounts on education as a share of GDP (OECD 2026). Moreover, Blanchet, Chancel, and Gethin (2022) and Bozio *et al.* (2024) show that European countries have less inequality than the U.S. not because they redistribute more but rather because they have more equal distributions before taxes and transfers (see Figure 1A), highlighting the importance of policies that shape the distribution of pre-tax-and-transfer incomes.

Second, the planner's objective in the Mirrleesian framework is equality of outcomes: welfare weights depend on marginal utilities of income, not on *why* individuals earn what they earn. In practice, the public appears to prioritize equality of opportunity over equality of outcomes, with greater concern about inequality that arises from circumstances (such as luck or family background) than from choices (such as effort). Using data from the General Social Survey, Figure 1B shows that 88% of Americans favor policies that promote equal opportunity over policies that promote equal outcomes. Similarly, Alesina, Stantcheva, and Teso (2018) find that concerns about unequal opportunities are broadly held and largely bipartisan in the United States and Western Europe: in each of the countries they study, more than 80% of respondents view unequal opportunities as a problem or a serious problem. Stantcheva (2021) reports that U.S. support for raising taxes is much greater when the revenue is spent on better schools for children from low-income families (predistribution that promotes equal opportunity) than on transfers to non-working adults (redistribution that promotes equal outcomes).

These departures from the Mirrleesian approach have been studied in two separate literatures, one empirical and one theoretical. On the empirical side, a large and rapidly growing literature in labor economics studies intergenerational income mobility as a measure of equality of opportunity (e.g., Solon

1992; Mazumder 2005; Chetty *et al.* 2014). Several experimental and quasi-experimental studies estimate the impacts of pre-market interventions on earnings in adulthood and rates of intergenerational mobility (e.g., Heckman *et al.* 2010; Chetty, Friedman, and Rockoff 2014; Kline and Walters 2016; Chetty *et al.* 2020; Barr and Gibbs 2022; Gray-Lobe, Pathak, and Walters 2023; Jackson and Mackevicius 2024). This literature sheds light on the determinants of economic mobility, but does not establish what the evidence implies for the optimal mix of redistribution and predistribution as it is not directly tied to a normative theory of optimal policy.

On the theoretical side, a long tradition in social choice and political philosophy has formalized equality of opportunity around two principles (Roemer 1998; Fleurbaey 2008; Fleurbaey and Maniquet 2011, 2018; Roemer and Trannoy 2015). According to the *compensation principle*, people should be compensated for unequal circumstances such as luck, genes, and family background. According to the *responsibility principle*, people should be held responsible for the consequences of their choices such as effort. This literature has made important conceptual progress in formalizing non-welfarist values, but applications of these concepts in economics have focused on the implications for tax and transfer policies (e.g., Lockwood and Weinzierl 2015; Saez and Stantcheva 2016), rather than predistributive instruments that target unequal opportunities directly.

In this paper, we connect these literatures by developing a normative framework that shows how measures of intergenerational mobility and estimates of the impacts of predistribution on earnings can inform the optimal mix of predistribution and redistribution when the objective is equality of opportunity. The goal of our framework is analogous to that of Diamond (1998) and Saez (2001), who showed how measures of income inequality and estimates of labor supply elasticities inform the design of redistributive policies when the objective is equality of outcomes.

Our framework departs from the canonical Mirrleesian approach in two ways. First, we allow labor market skill to depend on government investments made before individuals enter the labor market, which can be conditioned on family background (e.g., parental income). We refer to this policy instrument as "education," but it represents any policy that affects the mapping between family background and realized earnings. Second, we consider a government with preferences for equality of opportunity. To separate opportunities from outcomes, we assume that realized earnings depend on preferences for working (denoted by A) and family background (denoted by ω_0), both of which are heterogeneous in the population. The government respects inequality arising from differences in preferences (consistent with the responsibility principle), but seeks to reduce inequality arising from differences in family background (consistent with the compensation principle).[1] We capture these principles by specifying social welfare as a

[1]As discussed below, rather than labeling A and ω_0 as "preferences" and "family background," respectively, we could alternatively label them as meritocratic and unmeritocratic skill. The key assumption is that earnings are driven by two-

function of individual utilities evaluated at the *average* preferences in the population. Under this objective, higher earnings resulting from stronger preferences for work do not reduce an individual's social marginal welfare weight, whereas higher earnings resulting from a more advantaged family background do.[2] We establish conditions under which this social welfare function respects the Pareto principle, addressing classic concerns about non-welfarist social objectives (Kaplow and Shavell 2001*a*,*b*).

The government jointly chooses income tax rates and education provision to maximize its equal-opportunity objective. We show that the optimal tax schedule follows a generalized Diamond-Saez rule (Diamond 1998; Saez 2001; Diamond and Saez 2011), while the optimal education schedule follows a generalized Samuelson rule (Samuelson 1954).[3] The optimal tax rule depends on social marginal welfare weights measured at average preferences and the elasticity of earnings with respect to the marginal net-of-tax rate. The source of earnings inequality matters for optimal tax progressivity: the more that earnings inequality is driven by differences in family background, the more progressive the optimal tax schedule.[4] The optimal education rule equates the social marginal benefit and social marginal cost of education provision at each level of family background. The benefit is governed by the elasticity of earnings with respect to education spending, scaled by the direct welfare effect of higher skill and a fiscal externality of higher skill.

Our baseline model assumes that the government can target education directly to family background (perfect tagging) and that parental education investments are exogenous to government policy. We show that both of these assumptions can be relaxed without changing the core conclusions of our analysis. In particular, allowing for endogenous parental investments does not change the optimal policy rules in a reduced-form sense, but simply requires a reinterpretation of the relevant earnings elasticities. The tax rule depends on a macro elasticity that includes both the direct effect of children's supply responses to their marginal tax rate and the indirect effect of parents' investment responses to their children's marginal tax rate. The education rule depends on an earnings elasticity net of any crowding-out effects from parental responses to government policy. Reduced-form estimates of the impact of childhood interventions on adult earnings include such crowding-out and can therefore be used to calibrate our model.

To clarify, pre-market government investments have three advantages over taxes and transfers in our framework. First, they provide a targeting advantage by being conditioned on family-background skill

dimensional heterogeneity, where the government respects one dimension but not the other.

[2] A reduced-form alternative to our approach is to incorporate the equal-opportunity principles directly in the social marginal welfare weights (see, e.g., Lockwood and Weinzierl 2015; Saez and Stantcheva 2016). In our approach, the welfare weights are explicitly tied to (rescaled) individual utilities.

[3] The Samuelson rule in our setting accounts for the presence of distortionary taxation and distributional concerns (as in, e.g., Stiglitz and Dasgupta 1971; Atkinson and Stern 1974; Hylland and Zeckhauser 1979; Kaplow 1996, 2004; Kreiner and Verdelin 2012) and for the fact that education is a publicly provided *private* good.

[4] Conversely, if earnings inequality is unrelated to family background and driven entirely by preference heterogeneity, optimal marginal tax rates equal zero at *any* earnings elasticity.

ω_0 (perfect tagging) or parental earnings (imperfect tagging). For governments interested in equality of opportunity, this is preferred to income taxes based on current earnings regardless of their source.[5] Second, they generate public-goods benefits if government and parental education investments are imperfect substitutes, or if they are perfect substitutes but parents underinvest in their children relative to the social optimum. In these cases, the education good provided by the government is underprovided by the private market. Third, because education investments are complementary to labor supply, they violate the conditions of the Atkinson-Stiglitz theorem (Atkinson and Stiglitz 1976) under which governments should use only income taxes. In general, it is second-best optimal to either subsidize labor supply complements (Christiansen 1984; Saez 2002) or provide them in-kind (Nichols and Zeckhauser 1982).

To obtain further insight into the optimal mix of predistribution and redistribution, we simulate optimal tax and education policies using a calibrated version of our model. We calibrate the model to match empirical estimates of the average elasticity of adult earnings with respect to K-12 education spending based on Jackson and Mackevicius (2024) and Chetty, Friedman, and Rockoff (2014). Labor supply elasticities and other parameters are analogous to those used in prior work (e.g., Saez 2001).

We begin by analyzing the case with no preference heterogeneity, where our social welfare function coincides with the standard Mirrleesian objective. This case isolates the role of introducing predistribution. We find that the optimal education schedule is highly progressive: annual spending per child is nearly \$50,000 at the bottom of the family-background distribution and declines monotonically toward zero at the top. This predistribution relieves pressure on the tax-transfer system, substantially reducing marginal tax rates and the demogrant compared to the optimum without predistribution. We compare consumption inequality at the optimal policy and the laissez-faire equilibrium, decomposing the difference into the effects of predistribution and redistribution. Predistribution generates one-third of the reduction in the Gini coefficient and more than two-thirds of the reduction in the ratio of the 90th to the 10th percentile.[6]

We conduct comparative statics analyses of how this decomposition changes with key empirical parameters. The role of predistribution relative to redistribution increases with the earnings elasticity with respect to the net-of-tax rate and the earnings elasticity with respect to education, as these elasticities govern the relative efficiency of the two instruments. Heterogeneity in the education elasticity by family background is also important: optimal predistribution is larger when the effects of education are larger at the bottom than at the top of the family-background distribution. This comparative static highlights

[5] If income taxes could be conditioned on background skill or parental earnings, then the targeting advantage of pre-market investments would disappear. In fact, if taxes were allowed to depend on exogenous background skill ω_0, we would be in the first-best world of the second welfare theorem where governments can costlessly redistribute endowments (opportunities). We focus on governments setting taxes based solely on the current earnings of tax-liable individuals, because this is the universal form of redistribution across countries.

[6] The share attributable to predistribution is larger for the P90/P10 ratio than for the Gini coefficient as predistribution matters most at the bottom of the distribution.

the importance of estimating not only the average returns to education in the tradition of Mincer (1974), but also heterogeneity in returns by family background, as in the more recent literature on education and economic mobility (e.g., Zimmerman 2019; Chetty *et al.* 2020; Biasi, Lafortune, and Schönholzer 2025; Mountjoy 2026). Allowing for endogenous parental investments has little impact on the share of inequality reduction optimally achieved through predistribution.[7]

Next, we incorporate heterogeneity in preferences for work, in which case our equality-of-opportunity objective deviates from the Mirrleesian objective. To calibrate the share of earnings variation explained by preference heterogeneity vs family background, we build upon the empirical literature on intergenerational mobility and equality-of-opportunity measurement (e.g., Roemer and Trannoy 2016; Hufe *et al.* 2017; Hufe, Kanbur, and Peichl 2022). Using the 1979 National Longitudinal Survey of Youth (NLSY), we first show that approximately 10% of the variation in children's earnings in adulthood is explained by differences in parental income, consistent with estimates from modern administrative data (Chetty *et al.* 2014). We then expand the set of family background variables beyond parental income to include race, gender, parental education, and other variables measured before age 18 such as test scores, and show that this more expansive definition of family background explains 22.9% of the variation in children's earnings. An additional 18.5% of earnings variation is explained by variables that reflect choices made in adulthood (such as hours worked, educational attainment, and crime and health behaviors), leaving 58.6% of the variation unexplained by observable family background and choice variables.[8] Under a proportional benchmark that assigns the unexplained variation according to the explained shares, family background accounts for 55% of the variation in earnings and preference heterogeneity accounts for 45%. This degree of preference heterogeneity implies a substantially larger role for predistribution relative to redistribution than in the baseline case of homogeneous preferences. The reason is that preference heterogeneity strengthens the targeting benefit of providing education based on family background compared to taxing incomes that reflect both preferences and family background.

Our paper builds on and contributes to a large literature on optimal redistribution, including papers in the Mirrleesian tradition (e.g., Mirrlees 1971; Diamond 1998; Saez 2001; Piketty and Saez 2013) and papers that allow for non-welfarist fairness principles (e.g., Saez and Stantcheva 2016; Fleurbaey and Maniquet 2018). Related to our modeling of social welfare, Lockwood and Weinzierl (2015) develop a model with heterogeneity in both abilities and preferences, characterizing the optimal tax implications of preference-

[7] This holds even when public and private investments are close (possibly perfect) substitutes. The reason is that high social marginal welfare weights at the bottom combined with fiscal externalities via income taxation yield government education provision that far exceeds what parents would choose on their own.

[8] Any distinction between choices and family background requires judgments about free will; we treat choices made after age 18 as preference-related in our calibration, but these choices may themselves be driven by family background. Which sources of inequality warrant compensation ultimately depends on public judgments that could be elicited through surveys in future work, much like preferences for equality of opportunity themselves.

neutral welfare weights. Gelber and Weinzierl (2016) study optimal taxation in an intergenerational model where children's labor market abilities depend on parents' disposable incomes. In their model, income taxes achieve both redistribution and predistribution, thereby raising optimal tax progressivity. Berg, Håvarstein, and Stubhaug (2025) introduce social welfare functions that value reducing income inequality due to circumstances, but not income inequality due to merit, finding that optimal redistribution is considerably lower than in standard models. Our paper departs from this prior work by modeling the optimal mix of redistribution and predistribution and characterizing how this policy mix depends on equality-of-opportunity preferences.

A literature has investigated whether public provision of private goods can be justified on redistributive grounds. Arrow (1971) considered public expenditures targeted to individual types in a setting without income taxes or transfers. Subsequent research merged the analysis of public goods provision with the analysis of optimal income taxation (e.g., Ulph 1977; Blomquist and Christiansen 1995; Boadway and Marchand 1995; Pirttilä and Tuomala 2002). The main question in this literature is whether public provision can relax the incentive-compatibility constraints that restrict optimal redistribution. The results are related to the theory of taxing/subsidizing specific goods (Atkinson and Stiglitz 1976; Christiansen 1984; Saez 2002) or providing in-kind transfers (Nichols and Zeckhauser 1982). Closer to our agenda, Haaparanta *et al.* (2022) study optimal taxation and education provision under an equality-of-opportunity objective defined directly over the distribution of educational outcomes. Their analysis is restricted to linear income taxes and uniform education provision, whereas we analyze fully nonlinear policy instruments and allow education to be targeted by family background. We provide policy rules and quantitative analyses that are directly linked to the empirical literature on economic mobility.

Finally, several studies have extended the Mirrleesian framework to allow for human capital investments (e.g., Bovenberg and Jacobs 2005; Jacobs 2013; Stantcheva 2017). This body of work considers models where workers acquire human capital to increase their wages, and where the government jointly optimizes income taxes and education subsidies. In such models, income taxes lower the return to education and lead to underinvestment, making it socially optimal to subsidize education to restore efficiency and leave redistribution to the income tax instrument. The Bovenberg-Jacobs result prescribes full tax deductibility of human capital expenses, which is regressive by itself. In contrast to this literature, our model considers education investments made by parents and the government *pre-market*; there are no education investments made by adult workers. Government education provision creates equity gains by reducing inequality in pre-market skill, depending on the degree to which such provision crowds out parental investments. Therefore, while prior work and our paper both consider human capital

investments, they analyze different policy instruments (subsidies vs public provision) that diverge in terms of timing (investments by workers vs investments before labor market entry), targeting (own vs family-background characteristics), and underlying welfare motives (efficiency vs equity). Hence, their policy implications run in opposite directions: regressive subsidies paired with more redistributive taxes, as opposed to progressive public provision paired with less redistributive taxes.

The rest of the paper is organized as follows. Section 2 sets up our baseline model with fixed parental investments and perfect tagging of government investments. Section 3 extends the model to allow for endogenous parental investments and imperfect tagging. Section 4 presents numerical simulations based on empirically grounded calibrations. Section 5 concludes.

2 Baseline Model

2.1 Workers

A continuum of mass 1 of workers vary in terms of skill ω and preferences for working A. Consumption is given by $z - T(z)$, where z is earnings and $T(z)$ is a tax schedule. Preferences over consumption and effort are characterized by a quasi-linear utility function:

$$u = z - T(z) - \frac{A\omega}{1 + 1/\varepsilon} \left(\frac{z}{A\omega} \right)^{1+1/\varepsilon}. \tag{1}$$

The assumption of quasi-linearity eliminates income effects on effort and is common in the optimal tax literature (e.g., Diamond 1998; Kleven, Kreiner, and Saez 2009). The second term—disutility of effort—has two features worth noting: it is iso-elastic and it is scaled by $A\omega$. As we shall see, the scaling implies that $A\omega$ can be interpreted as a measure of "earnings capacity." This form of scaling has been used in a number of papers (Kleven, Kreiner, and Saez 2009; Saez 2010; Kleven *et al.* 2025; Kleven 2026), albeit in models without preference heterogeneity operating through A. The parameterization in equation (1) is analytically convenient and useful for interpretation, but it is not critical for any of the qualitative results presented below.

The maximization of utility with respect to earnings gives the following labor supply function

$$z = A\omega \left(1 - \tau(z)\right)^{\varepsilon}, \tag{2}$$

where $\tau(z) \equiv T'(z)$ is the marginal tax rate, and ε is the elasticity of earnings with respect to the marginal net-of-tax rate $1 - \tau$. Equation (2) implies that, at a marginal tax rate of zero, workers choose $z = A\omega$. In other words, the product of preferences and skill can be interpreted as laissez-faire earnings or earnings

capacity. The imposition of positive marginal tax rates depresses actual earnings z below earnings capacity $A\omega$ according to the elasticity parameter ε.

With a slight abuse of notation, indirect utility can be written as $u(A\omega)$. In a standard Mirrleesian framework, the planner's social welfare function is defined over $u(A\omega)$ across workers of different types. In this case, the distinction between A and ω—given they enter symmetrically—would be irrelevant. But we consider a planner who evaluates inequality driven by A and ω differently, respecting the former but not the latter. As described below, our social objective provides a tractable way of capturing the principles of *compensation* and *responsibility* from the literature on equality of opportunity (Fleurbaey and Maniquet 2011; Roemer and Trannoy 2015; Fleurbaey and Maniquet 2018). While we label A as preferences and ω as skill, we could alternatively label them as meritocratic and unmeritocratic skill, respectively. Regardless of labels, the substantive assumption is that earnings inequality is driven by two dimensions of heterogeneity, where the government views only one of them (the "opportunity") as worthy of redistribution. The income tax $T(z)$ does not distinguish between the two, blunting its effectiveness for redistributing in a welfare-relevant manner.

2.2 Skill Production

The skill parameter ω is not a primitive of the model, in contrast to Mirrleesian models where skill reflects innate ability. Instead, we assume that skill depends partly on family background (parental skill) and partly on a government instrument such as education provision (predistribution).

The production function for skill takes the following form

$$\omega = \Omega(\omega_0, E(\omega_0)) \equiv \widehat{\Omega}(\omega_0). \tag{3}$$

Realized skill ω is governed by two factors. The first is background skill ω_0 drawn from an exogenous distribution—the effect of family background. Realized skill is non-decreasing in background skill, $\Omega'_{\omega_0} \geq 0$. The second is government investment in education $E(\omega_0)$—the effect of predistribution. These investments are made prior to the labor supply choices in equation (2) and they can be conditioned on family background ω_0.[9] Realized skill is also non-decreasing in education, $\Omega'_E \geq 0$. Our analytical characterizations do not rely on any specific functional form for $\Omega(.)$, but we assume that, at the optimal education schedule, the total derivative of realized skill ω with respect to background skill ω_0 is non-negative, i.e., $\widehat{\Omega}'(\omega_0) \equiv \Omega'_{\omega_0} + \Omega'_E \cdot E'(\omega_0) \geq 0$. In other words, there is intergenerational persistence in the model, even after accounting for predistribution.

[9] In practice, public education provision depends on family-background variables such as location and parental income, which map into a relationship between children's education and parental skill ω_0. We consider a government that chooses a flexible and potentially nonlinear education schedule $E(\omega_0)$ as a function of parental skill.

The production function in equation (3) has three features worth emphasizing. First, it assumes that the transmission of background skill ω_0 from parents to children is exogenous. Parents do not make private education investments in their children, or at least none that are endogenous to government policies. We generalize the model to permit endogenous parental education investments in Section 3.1.

Second, publicly provided education is modeled as a private good. It is both excludable and rival. This implies that education can be targeted to specific individuals without any productivity spillovers to other individuals. Our baseline model assumes perfect tagging to family background. We generalize the model to permit imperfect tagging in Section 3.2.

Third, our specification differs from prior work extending the Mirrleesian framework to allow for human capital investments (e.g., Bovenberg and Jacobs 2005; Stantcheva 2017). This literature considers models where adult workers acquire human capital to increase their wages, and where the government may tax or subsidize those human capital investments in addition to taxing income. A key insight from that literature is that governments should generally subsidize human capital investments to offset the disincentives created by redistributive income taxation. This is an efficiency argument. Optimal education subsidies are typically regressive, while the full system of income taxes and education subsidies is progressive.[10] In our baseline model, there are no private education investments—and therefore no income-tax distortion of those investments—but rather public education investments made pre-market. A progressive education schedule creates equity gains by reducing inequality in pre-market skill.[11] With an equal-opportunity objective, the equity gains of early-education provision are larger than the equity gains of income taxes—provided the two are equally progressive—as the former redistributes pre-market skill while the latter redistributes income driven by both pre-market skill and preferences.

2.3 Policy Instruments and Levels of Inequality

The model incorporates four levels of inequality, linked by government policies. The first level is heterogeneity in innate types—background skill ω_0 and preferences for working A. The joint distribution of innate types is given by a density function $f_0(\omega_0|A) f_A(A)$. The second level is heterogeneity in realized types (ω, A), which depends on government investments in education. Given the skill production function (3), the joint distribution of realized types is given by the density function $f_\omega(\omega|A) f_A(A) = f_0\left(\widehat{\Omega}^{-1}(\omega)|A\right) \frac{\partial \widehat{\Omega}^{-1}(\omega)}{\partial \omega} f_A(A)$. The difference between the innate and realized type

[10] A classic result is that, in a static and deterministic model of human capital acquisition, the optimal policy makes human capital expenses fully tax deductible (Bovenberg and Jacobs 2005). With progressive marginal income tax rates, education subsidies are regressive.

[11] Education provision has efficiency effects as well, both through fiscal externalities and through public goods benefits. The latter effect arises because, in this model, there is no private market for the education good provided by the government. Even with a private education market (Section 3), there will be public goods benefits if private and public education are not perfect substitutes or if parents underinvest in their children relative to the social optimum.

distributions represents *predistribution*. The third level is heterogeneity in before-tax earnings z, which depends on marginal income tax rates. Given the labor supply function (2), the joint distribution of earnings and preferences is given by $f_z(z|A) f_A(A) = f_\omega\left(\frac{z}{A(1-\tau(z))^\varepsilon}|A\right)\frac{1}{A(1-\tau(z))^\varepsilon} f_A(A)$. The final level is inequality in after-tax earnings $z - T(z)$, determined by the distribution of before-tax earnings (which depends on marginal tax rates) and the tax schedule $T(z)$. The difference between the distributions of realized types and after-tax earnings represents *redistribution*.

Our goal is to characterize the tradeoff between predistribution (equalizing pre-market opportunities) and redistribution (equalizing post-market outcomes) using formulas that highlight the key mechanisms and depend on estimable sufficient statistics.

2.4 Government's Objective

We consider a government with preferences for equality of opportunity. A rich social choice literature studies distributional objectives that incorporate fairness principles beyond the standard utilitarian perspective of public economics (e.g., Roemer 1998; Fleurbaey and Maniquet 2011, 2018; Roemer and Trannoy 2015; Saez and Stantcheva 2016). In this literature, the equal-opportunity ethic is typically based on the principles of compensation and responsibility. According to the *compensation principle*, people should be compensated for unequal circumstances such as luck, genes, and family background. According to the *responsibility principle*, people should be held responsible for unequal choices such as effort. Our social objective is grounded in these ideas, but formalized in a novel way.

The equal-opportunity ethic just described, while intuitive, must address an important challenge: how do we separate circumstances and choices? This is not only an empirical challenge but a conceptual one, because choices are functions of circumstances. In the Mirrleesian framework, workers are born with an innate ability (circumstance) which determines labor supply (choice); there is no meaningful sense in which choices can be separated from circumstances. This is true of all economic models that characterize choices in terms of primitives and external factors such as preference parameters, market prices, government policies, and stochastic processes—none of which are chosen by the individual.[12]

To address these conceptual issues, we specify the equal-opportunity objective in terms of primitives that can be clearly separated: background skill ω_0 and preferences for working A. The social planner respects inequality driven by preferences A (responsibility principle), but not inequality driven by family background ω_0 (compensation principle).[13] While we label the two primitives as preferences and

[12] A more philosophical perspective on this point is that, in economic models, there is no free will. Economic agents solve decision-making problems, but these are essentially translations of external factors into choices. The difference between any two agents can be explained entirely by differences in factors outside the agent's span of control.

[13] This definition of equal-opportunity preferences is robust to the absence of free will.

background skill, we could alternatively label them as meritocratic and unmeritocratic skill.[14] The fact that they enter the worker's problem symmetrically (via earnings capacity $A\omega$) is analytically convenient, but not important. The key assumption is that earnings capacity depends on two factors, only one of which merits redistribution.[15]

To incorporate the idea that redistribution in the preference dimension has no social value, we assume that the government evaluates individual utilities at the *average* preferences for working in the population. Specifically, the social welfare function is given by

$$\begin{aligned} W &= \int_A \int_{\omega_0} \Psi \left[u \left(z \left(A^* \widehat{\Omega} \left(\omega_0 \right) \right), A^* \widehat{\Omega} \left(\omega_0 \right) \right) \right] dF_0 \left(\omega_0 | A \right) dF_A \left(A \right) \\ &= \int_A \int_z \Psi \left[u \left(\frac{A^*}{A} z, A^* \omega \left(z, A \right) \right) \right] dF_z \left(z | A \right) dF_A \left(A \right), \end{aligned} \tag{4}$$

where $\Psi \left[. \right]$ is an increasing and concave transformation of individual utilities, and $A^* \equiv \int_A A dF_A \left(A \right)$ denotes average preferences for working. The first line of equation (4) is specified in terms of background skill ω_0, while the second line is specified in terms of earnings z. The second line clarifies that, by valuing each individual at average preferences, the planner scales their earnings by a factor A^* / A, treating those with strong (weak) preferences for working as if they had lower (higher) earnings than they actually do.[16]

When setting taxes and education provision, the government must satisfy the following budget constraint

$$\begin{aligned} \int_A \int_{\omega_0} T \left(z \left(A \widehat{\Omega} \left(\omega_0 \right) \right) \right) dF_0 \left(\omega_0 | A \right) dF_A \left(A \right) - p \int_A \int_{\omega_0} E \left(\omega_0 \right) dF_0 \left(\omega_0 | A \right) dF_A \left(A \right) &\geq R \iff \\ \int_A \int_z T \left(z \right) dF_z \left(z | A \right) dF_A \left(A \right) - p \int_A \int_{\omega_0} E \left(\omega_0 \right) dF_0 \left(\omega_0 | A \right) dF_A \left(A \right) &\geq R, \end{aligned} \tag{5}$$

where p is the price per unit of education, and R is an exogenous revenue requirement (for financing non-education spending). We denote the Lagrange multiplier associated with the government budget constraint by λ. Note that this multiplier corresponds to the Marginal Value of Public Funds (MVPF).

The government sets the tax schedule $T \left(z \right)$ and the education schedule $E \left(\omega_0 \right)$ to maximize social

[14] With only two goods—consumption and leisure—the parameter A determines the relative weighting of consumption and leisure in utility. Therefore, preferences for working A could also be labeled as preferences for consuming. In this terminology, the equal-opportunity objective puts a zero value on income inequality driven by heterogeneity in consumption preferences.

[15] This approach is closely related to that of Lockwood and Weinzierl (2015). Note that the symmetric formulation, together with the fact that the tax system depends only on earnings z, implies that each worker's marginal rate of substitution between consumption and earnings depends on A and ω only through the composite object $A\omega$. Therefore, standard single-crossing and unidimensional screening arguments apply despite the two-dimensional heterogeneity. The only implication of multidimensional heterogeneity is a composition effect: conditional on earnings, there is a distribution of social marginal welfare weights that must be properly averaged, as we show when characterizing optimal policy below. Jacquet and Lehmann (2021) provide a general analysis of such composition effects in optimal tax formulas.

[16] When expressing social welfare in terms of earnings z, $\omega \left(z, A \right) \equiv \frac{z}{A(1-\tau(z))^{\varepsilon}}$ defines the skill level ensuring that a worker with preferences A chooses z given the labor supply function in equation (2).

welfare (4) subject to incentive compatibility constraints (2), the skill production function (3), and the government budget constraint (5). The tax and education schedules may be nonlinear, but they are assumed to be continuously differentiable.

Social Marginal Welfare Weights: The government's redistributive preferences can be summarized by social marginal welfare weights: the social marginal value of income to different individuals ($\Psi'[.]$) in terms of the marginal value of public funds (λ). These welfare weights can be expressed in terms of background skill ω_0, realized skill ω, or earnings z, using the mapping between the three concepts provided by our model. For characterizing optimal income taxes $T(z)$, it is convenient to express welfare weights in terms of earnings. We define $g(z, A, A^*) \equiv \Psi'\left[u\left(\frac{A^*}{A}z, A^*\omega(z, A)\right)\right]/\lambda$ as the social marginal welfare weight on individuals with earnings z and preferences A. This allows us to specify average welfare weights as follows:

$$g(z, A^*) \equiv \frac{\int_A g(z, A, A^*) f_z(z|A) dF_A(A)}{h(z)} \tag{6}$$

$$G(z, A^*) \equiv \frac{\int_z^{\bar{z}} g(z', A^*) dH(z')}{1 - H(z)}, \tag{7}$$

where $h(z) \equiv \int_A f_z(z|A) dF_A(A)$ and $H(z) \equiv \int_A F_z(z|A) dF_A(A)$ denote the *unconditional* density and distribution functions for earnings.[17] These equations define average social marginal welfare weights at earnings level z ($g(z, A^*)$) and above earnings level z ($G(z, A^*)$), each measured at average preferences A^*. With government preferences for equality, the weights g, G will in general be declining in z. However, because we hold preferences fixed at their average, the weights will decline less steeply than in standard models whenever some of the earnings inequality reflects preference heterogeneity. If earnings inequality were driven solely by preference heterogeneity, the welfare weights would be flat in z. In the optimal policy analysis, the equal-opportunity objective is fully captured by the welfare weights defined above.

Our approach is consistent with the argument in Saez and Stantcheva (2016) that different fairness principles can be captured by generalized social marginal welfare weights. The key difference between the two approaches is that, in our framework, the welfare weights are explicitly tied to individual utilities.

Pareto Principle: A concern with social welfare functions that include fairness principles beyond those based on individual utilities is that they may be non-Paretian (Kaplow and Shavell 2001*a*,*b*), i.e., there may be policy changes that make all individuals (weakly) better off but nevertheless generate lower social

[17] In equation (7), we denote the upper bound of the earnings distribution by $\bar{z}$. With an infinite upper tail, we have $\bar{z} = \infty$.

welfare. The following remarks establish conditions under which the social welfare function in equation (4) respects the Pareto principle.

Remark 1 (**Utility Perturbations and the Pareto Principle**). *Given a distribution of primitives* (A, ω_0)*, any set of individual-specific utility perturbations such that no individual is worse off (better off) and at least one individual is strictly better off (worse off) implies a strictly larger (smaller) value of the social welfare function* W *defined in equation (4). Because of quasi-linear utility, these utility perturbations correspond to individual lump-sum taxes or transfers based on type.*

Proof. See Appendix C. □

This remark shows that when allowing for individual lump-sum taxes and transfers, our social welfare function does not generate non-Paretian policy recommendations. This result relies only on $\Psi[.]$ being strictly increasing, as these lump-sum instruments affect realized and imputed utilities in the same direction.

It is worth noting that the result holds the distribution of primitives fixed. If not, there is a trivial sense in which non-Paretian results are achievable, namely if we allow the planner to manipulate the structure of primitives. For example, suppose all heterogeneity in earnings capacity is due to family background ω_0 rather than preferences A. If the planner shifts heterogeneity from ω_0 to A such that all individual utilities are kept constant, the value of social welfare W increases. Then, if the planner destroys a small amount of A for one individual, social welfare remains higher than in the initial equilibrium even though the change in allocation is a Pareto worsening. More generally, we show in Appendix C that, in the setting with individual lump-sum instruments, *any* change in allocation associated with non-Paretian effects must involve a reallocation of the primitives of the model. Such non-Paretian effects have no policy relevance as they are not grounded in any feasible instrument at the government's disposal.

Remark 1 is insufficient to assess whether the social objective (4) is Paretian under our restricted policy environment. The instruments $T(z)$ and $E(\omega_0)$ are targeted at different variables, so they can have non-monotone effects on the gap between realized and imputed utilities. However, under mild regularity conditions, the optimal policy is always Pareto efficient.

Remark 2 (**Optimal Policy and the Pareto Principle**). *Assume that* $A^* \in \text{supp}(A|\omega_0)$ *for all* ω_0*. Then, the social welfare function* W *defined in equation (4) is weakly Paretian: any allocation induced by* $T(z)$ *and* $E(\omega_0)$ *where all individuals are weakly better off yields a weakly larger value of the social welfare function* W*. Moreover, if* $\Omega(\omega_0, E(\omega_0))$ *is strictly concave in* $E(\omega_0)$ *and under a set of regularity conditions (the support of* ω_0 *is connected;* $A^* \in \text{int supp}(A|\omega_0)$ *for all* ω_0*; the individual problem admits a unique solution in* z*; and* $\widehat{\Omega}(\omega_0)$ *is continuous*

and strictly positive for all ω_0), then the policies $T(z)$ and $E(\omega_0)$ always induce a strictly Pareto efficient allocation at the social optimum.

Proof. See Appendix C. □

To understand Remark 2, consider a situation where, for some background type ω_0, A^* is not part of the support of A. In that case, the imputed utility $u\left(A^*\widehat{\Omega}(\omega_0)\right)$ does not correspond to the utility of any actual individual in the economy. When social welfare depends on such "artificial" individuals, it can be optimally manipulated in ways that violate the Pareto principle. Assuming that $A^* \in \text{supp}(A|\omega_0)$ for all ω_0 grounds the reference point in real individuals, preventing the planner from engaging in "creative" policy functions that increase social welfare by manipulating the utilities of artificial individuals in ways that hurt real individuals.

This notion of Pareto efficiency is not strict: there could be cases where all individuals are weakly better off, one individual is strictly better off, but social welfare is unchanged. In Appendix C we characterize those "non-Paretian ties" and show that, under the regularity conditions specified in Remark 2, these are not relevant at the social optimum. Intuitively, non-strictly Paretian policies leave money on the table: if one individual is strictly better off but social welfare is unchanged, then there is always an alternative policy that is socially preferred. Hence, any violations of the strict Pareto principle allowed by our social welfare function do not affect the optimal policies characterized below.

2.5 Optimal Policy

We first characterize the optimal tax schedule (redistribution of income), taking the education schedule (predistribution of skill) as given. We solve for the optimal marginal tax rates using a perturbation approach (Saez 2001). This approach is based on the idea that, at the optimum, small changes in the tax schedule have no first-order effects on welfare.

Consider a small reform which increases the marginal tax rate by $d\tau$ on an earnings interval $(z, z+dz)$. Such a reform has three welfare effects: a mechanical revenue effect dM_τ, a behavioral revenue effect dB_τ, and a direct welfare effect dW_τ. We derive each of these effects below, characterizing the optimal tax schedule based on the condition $dM_\tau + dB_\tau + dW_\tau = 0$.

The mechanical revenue effect (evaluated at the marginal value of public funds λ) is given by

$$dM_\tau = \lambda \cdot d\tau \cdot dz \cdot (1 - H(z)). \tag{8}$$

This expression captures that all workers above earnings level z (with mass $1 - H(z)$) experience a tax liability increase of $d\tau \cdot dz$, taking behavior as given.

The behavioral revenue effect (also evaluated at the marginal value of public funds λ) is given by

$$\begin{aligned} dB_\tau &= \lambda \cdot \tau(z) \cdot dz \cdot h(z)\, dz \\ &= -\lambda \cdot \frac{\tau(z)}{1-\tau(z)} \cdot \varepsilon \cdot z \cdot d\tau \cdot h(z)\, dz, \end{aligned} \tag{9}$$

where we use the earnings supply function (2). This expression captures that workers in the affected range (with mass $h(z)\, dz$) reduce earnings by dz and therefore tax liability by $\tau(z) \cdot dz$. This is the fiscal externality—or efficiency effect—from behavioral responses.

Finally, the direct welfare effect can be written as

$$\begin{aligned} dW_\tau &= -\int_A \int_z^{\overline{z}} \Psi'[.] \cdot d\tau \cdot dz \cdot dF_z(z'|A)\, dF_A(A) \\ &= -\lambda \cdot d\tau \cdot dz \cdot (1 - H(z)) \cdot G(z, A^*) \\ &= -dM_\tau \cdot G(z, A^*), \end{aligned} \tag{10}$$

where we use the definition of $G(z, A^*)$ in equation (7). These derivations allow us to characterize the optimal tax schedule as follows:

Proposition 1 (Optimal Redistribution). *For any given education schedule $E(\omega_0)$, the optimal marginal tax rate at each earnings level z satisfies*

$$\frac{\tau(z)}{1-\tau(z)} = \frac{1 - G(z, A^*)}{\varepsilon \cdot \alpha(z)}, \tag{11}$$

where ε is the elasticity of earnings with respect to $1-\tau$, $\alpha(z) \equiv zh(z) / (1 - H(z))$ denotes the local Pareto parameter of the earnings distribution, and $G(z, A^*)$ denotes the average social marginal welfare weight above earnings level z, measured at average preferences A^*, as defined in equation (7).

Proof. The result in equation (11) follows from the optimality condition $dM_\tau + dB_\tau + dW_\tau = 0$, using the characterization of each term in equations (8)-(10). We call $\alpha(z)$ a local Pareto parameter because, for an exact Pareto distribution with parameter a, we have $\alpha(z) = a$. □

Formally, the only difference between our optimal tax rule and the Mirrlees-Diamond-Saez rule (Mirrlees 1971; Diamond 1998; Saez 2001) lies in the social marginal welfare weight $G(z, A^*)$. Our model implies a different profile of welfare weights because of two features: (*i*) earnings inequality is driven by heterogeneity in both background skill and preferences, and (*ii*) the government has an equal-opportunity objective that respects inequality driven by preferences (responsibility principle). All else equal, this flattens the profile of welfare weights, reducing the progressivity of marginal tax rates.[18]

[18] These remarks highlight that, in our model, the source of earnings inequality is critical for optimal taxation. In the special

A second difference between our model and the Mirrleesian model—not directly visible in the reduced-form representation of equation (11)—is the presence of a predistribution instrument. This instrument allows the government to reduce inequality in pre-market skill and therefore in earnings, feeding back into the optimal tax rule via $\alpha(z)$ and $G(z, A^*)$. This effect further reduces the progressivity of marginal tax rates. Importantly, while our model limits the desirability of traditional redistribution, optimal policy is not necessarily less progressive overall due to the role of predistribution.

We now turn to the characterization of the optimal education schedule (predistribution). As with taxes, the derivations are based on a perturbation approach. Consider a small reform which increases education by dE on a skill interval $(\omega_0, \omega_0 + d\omega_0)$. The reform has three welfare effects: a mechanical revenue effect dM_E, a behavioral revenue effect dB_E, and a direct welfare effect dW_E. At the social optimum, we must have $dM_E + dB_E + dW_E = 0$.

The mechanical revenue effect is given by

$$dM_E = -\lambda \cdot p \cdot dE \cdot \int_A f_0(\omega_0|A)\, d\omega_0 dF_A(A). \tag{12}$$

The behavioral revenue effect is given by

$$dB_E = \lambda \cdot \Omega'_E \cdot dE \cdot \int_A \tau(z) \frac{dz}{d\omega} \cdot f_0(\omega_0|A)\, d\omega_0 dF_A(A). \tag{13}$$

This expression characterizes the fiscal externality that arises because affected workers increase earnings and therefore tax liability (by $\tau(z)\frac{dz}{d\omega}$) when their skill increases (by $\Omega'_E(.) \cdot dE$) due to education provision.

Finally, the direct welfare effect is given by

$$\begin{aligned} dW_E &= \Psi'[.] \cdot \frac{du}{d\omega} \cdot \Omega'_E \cdot dE \cdot \int_A f_0(\omega_0|A)\, d\omega_0 dF_A(A) \\ &= \lambda \cdot g(\omega_0, A^*) \cdot \frac{du}{d\omega} \cdot \Omega'_E \cdot dE \cdot \int_A f_0(\omega_0|A)\, d\omega_0 dF_A(A), \end{aligned} \tag{14}$$

where $g(\omega_0, A^*) \equiv \Psi'\left[u\left(z\left(A^*\widehat{\Omega}(\omega_0)\right), A^*\widehat{\Omega}(\omega_0)\right)\right]/\lambda$ denotes the social marginal welfare weight on individuals with background skill ω_0, measured at average preferences A^*.

With these derivations, the optimal education schedule can be characterized as follows:

case where earnings inequality is driven entirely by preference heterogeneity, all workers get the same welfare weight g^*. Because the average welfare weight in the population equals one (this follows from the optimization of the intercept $T(0)$), we obtain $G(z, A^*) = g^* = 1$ and therefore $\tau(z) = 0$ at any z. This holds even as the earnings elasticity ε approaches zero. In the limit where both $G(z, A^*) = 1$ and $\varepsilon = 0$, social welfare is independent of marginal tax rates: they have neither equity benefits nor efficiency costs. In this case, the optimal tax schedule becomes indeterminate.

Proposition 2 (Optimal Predistribution). *For any given tax schedule $T(z)$, the optimal education provision at each level of background skill ω_0 satisfies*

$$\Omega'_E \cdot \left[g(\omega_0, A^*) \frac{du}{d\omega} + \mathbb{E}\left[\tau(z) \frac{dz}{d\omega} \mid \omega_0 \right] \right] = p, \tag{15}$$

where $\frac{du}{d\omega} = A^* (1 - \tau(z))^{1+\varepsilon} / (1+\varepsilon)$ denotes the marginal willingness to pay for higher skill, and $g(\omega_0, A^*)$ denotes the social marginal welfare weight at background skill ω_0, measured at average preferences A^*.

Proof. At the social optimum, we have $dM_E + dB_E + dW_E = 0$. Inserting equations (12)-(14) and rearranging terms gives equation (15). The formula for the marginal willingness to pay $\frac{du}{d\omega}$ follows from the utility specification (1), applying the labor supply function (2) (envelope theorem). □

Education provision is set to ensure that, at each skill level, the social marginal benefit (left-hand side of equation 15) equals the social marginal cost (right-hand side of equation 15). The social marginal benefit depends on the skill-effect of education, Ω'_E, scaled by two terms: a direct welfare effect of higher skill (first term) and a fiscal externality of higher skill (second term).

This result relates to the Samuelson rule for public goods provision (Samuelson 1954) in settings with distortionary taxation and distributional concerns (e.g., Stiglitz and Dasgupta 1971; Atkinson and Stern 1974; Hylland and Zeckhauser 1979; Kaplow 1996, 2004; Kreiner and Verdelin 2012). Two differences between the optimal education rule (15) and standard results are worth noting. First, our policy rule considers public provision of a private good (excludable and rival). The social marginal benefit of providing education to a specific group of individuals includes effects only for that group, as opposed to effects in the entire population. Second, our policy rule allows for education to be targeted based on family background ω_0 (tagging). The education schedule $E(\omega_0)$ may be progressive and nonlinear.

An alternative interpretation of Proposition 2 notes that education provision is a form of in-kind transfer, which increases skill and is complementary to labor supply. Hence, our result also relates to the literature on in-kind transfers (Nichols and Zeckhauser 1982) or subsidies to specific goods (Atkinson and Stiglitz 1976; Christiansen 1984; Saez 2002). This literature shows that labor supply complementarity creates an argument for in-kind transfers or subsidization. In our model, the degree of complementarity between education and labor supply is governed by Ω'_E. If $\Omega'_E = 0$, equation (15) implies zero education provision.

Because of the government's equal-opportunity objective, the source of earnings inequality is critical for both redistribution and predistribution. It is instructive to consider the special case where earnings inequality is driven entirely by preference heterogeneity:

Corollary 1 (Pure Preference Heterogeneity). *Suppose earnings inequality is driven only by preference heterogeneity. In this case, all workers have the same social marginal welfare weight* $(= 1)$ *such that* $G(z, A^*) = 1$. *The optimal tax schedule has* $\tau(z) = 0$ *at any* z, *and the optimal education schedule at background skill* ω_0 *satisfies*

$$\frac{A^* \cdot \Omega'_E}{1+\varepsilon} = p. \tag{16}$$

If skill production is separable in family background ω_0 *and education* E (*i.e.*, $\Omega''_{E,\omega_0} = 0$), *education provision is constant across workers. This corresponds to universal public education, financed by a lump-sum tax,* $T(z) = \overline{T}$.

In this special case, optimal tax rates are zero and education provision is motivated solely by efficiency considerations as there is no welfare-relevant inequality. The education instrument has efficiency effects (even absent fiscal externalities), because it compensates for a missing market: our baseline model assumes that households cannot purchase the same education good in the private market. Even with a private education market (Section 3), unless public and private education are perfect substitutes, there will be efficiency effects of public education.

Note that Ω'_E can be inferred from reduced-form empirical estimates of the effects of education on earnings (as we demonstrate in Section 4). Using the earnings supply function (2), the elasticity of earnings with respect to education spending is given by

$$\zeta \equiv \frac{dz/z}{dE/E} = \Omega'_E \cdot \frac{E}{\omega}. \tag{17}$$

In general, the education elasticity ζ varies by the level of skill and therefore earnings, depending on the shape of $\Omega(.)$. For example, if $\Omega(.)$ is separable in background skill and education (such that Ω'_E is independent of skill), ζ is a decreasing function of ω.[19] Equation (17) shows that, in this case, ζ goes from infinity ($\omega = 0$) to zero ($\omega = \infty$) as skill rises. In Section 4, we calibrate $\Omega(.)$ to match empirical estimates of the average education elasticity $\mathbb{E}[\zeta]$. Intuitively, the optimal mix of predistribution and redistribution depends on the relative magnitudes of tax and education elasticities, ε and $\mathbb{E}[\zeta]$.

To summarize, the optimal design of tax and education policies—redistribution and predistribution—is governed by equations (11) and (15). These policy rules provide intuition and highlight the central empirical parameters, but they are not closed-form solutions. To make explicit policy statements, the optimality conditions must be solved simultaneously using calibrated parameters and numerical simulations.

[19] In practice, arguments can be made for either complementarity or substitutability between inputs in skill production. Children with high family-background skill may be better able to acquire knowledge through schooling (implying $\Omega''_{E,\omega_0} > 0$), but they also have less to learn in school (implying $\Omega''_{E,\omega_0} < 0$). If these mechanisms offset each other, the reduced form $\Omega(.)$ would be separable in family-background skill ω_0 and education investments E.

3 Extensions

3.1 Allowing for Parental Education Investments

In our baseline model, parents transmit background skill ω_0 to their children in an exogenous manner. This intergenerational transmission may require parental education investments, but these were implicit and exogenous to government policy and children's behavior. We now generalize the model to allow parents—generation zero—to purchase an education good that increases the earnings capacity of their children. We choose units such that the prices of private education (E_p) and government education (E_g) are both equal to p.

The utility of parents with skill ω_0 is specified as

$$u_0 = z_0 - T_0(z_0) - pE_p - \frac{A_0\omega_0}{1+1/\varepsilon}\left(\frac{z_0}{A_0\omega_0}\right)^{1+1/\varepsilon} + \kappa \cdot u(A\omega), \tag{18}$$

where $u(A\omega)$ is the children's indirect utility and κ is the weight placed by parents on their children. A value of $\kappa = 1$ ($\kappa = 0$) corresponds to full (zero) internalization of the children's welfare. As shown above, the impact of higher skill on children's utility equals $\frac{du(A\omega)}{d\omega} = A(1-\tau(z))^{1+\varepsilon} / (1+\varepsilon)$. Skill production takes the following form

$$\omega = \Omega(\omega_0, E_p, E_g), \tag{19}$$

with derivatives $\Omega'_p, \Omega'_g \geq 0$ and $\Omega''_{pp}, \Omega''_{gg} \leq 0$. We impose no restrictions on the sign of the cross-derivative, Ω''_{pg}, allowing private and public investments to be either technological complements or substitutes.

Parents choose $E_p \geq 0$ to maximize utility (18) subject to skill production (19), having perfect information about government policies in the children's generation. When an interior solution exists, parental education investments satisfy the following first-order condition

$$\left[\frac{\kappa A(1-\tau(z))^{1+\varepsilon}}{1+\varepsilon}\right] \cdot \Omega'_p = p, \tag{20}$$

where the left-hand side measures the marginal benefit of parental education investments: the impact on children's skill, Ω'_p, scaled by the parents' marginal willingness to pay for their children's skill. Note that, as $\kappa \to 0$, the left-hand side goes to zero and the first-order condition has no interior solution with $E_p > 0$. In this case, parents choose $E_p = 0$ and all of our previous results apply.

From equation (20), we may characterize how parents respond to changes in government policy. We

obtain

$$\frac{dE_p}{d\tau} = \frac{\Omega'_p}{\Omega''_{pp}} \cdot \frac{1+\varepsilon}{1-\tau} \tag{21}$$

$$\frac{dE_p}{dE_g} = -\frac{\Omega''_{pg}}{\Omega''_{pp}}. \tag{22}$$

The sign of $dE_p/d\tau$ is negative—parents reduce their education investments when a higher tax rate makes those investments less valuable to the children—while the sign of dE_p/dE_g is determined by Ω''_{pg}. When private and public investments are independent in skill production, $\Omega''_{pg} = 0$, parents do not respond to changes in government education provision.

We consider a government maximizing social welfare in the children's generation, taking policies in the parents' generation as given.[20] Hence, the government's optimization problem takes the same general form as before, reinterpreting the revenue requirement R as being net of any leftover revenue from generation zero. The only difference between the optimization problems lies in the presence of the additional incentive compatibility constraint (20). This constraint implies that, whenever the government changes taxes or education provision for the children, parents change their education investments according to equations (21)-(22).

Consider first the optimal tax schedule. As before, we solve for the optimal schedule using a perturbation approach, increasing the marginal tax rate by $d\tau$ on an earnings interval $(z, z+dz)$. The three welfare effects dM_τ, dB_τ, and dW_τ characterized in equations (8)-(10) continue to apply, but there is an additional behavioral effect, dB^p_τ, because of parental investment responses to a higher tax rate on their children. This effect may be written as

$$\begin{aligned} dB^p_\tau &= \lambda \int_A \tau(z) \cdot \frac{dz}{d\omega} \cdot \frac{d\omega}{dE_p} \cdot dE_p \cdot f_z(z|A) \cdot dz \cdot dF_A(A) \\ &= -\lambda \cdot \frac{\tau(z)}{1-\tau(z)} \cdot \varepsilon_p \cdot z \cdot d\tau \cdot h(z) \cdot dz, \end{aligned} \tag{23}$$

where ε_p denotes an indirect earnings elasticity with respect to $1-\tau$, operating through parental education investments. It is defined as follows:

$$\varepsilon_p \equiv \mathbb{E}\left[\frac{dz/z}{dE_p/E_p} \frac{dE_p/E_p}{d(1-\tau)/(1-\tau)} \mid z\right] = -\mathbb{E}\left[\frac{(\Omega'_p)^2}{\Omega''_{pp}} \frac{1+\varepsilon}{\omega} \mid z\right] > 0. \tag{24}$$

The relationship between the reduced-form elasticity ε_p and model primitives—as shown in the last

[20] Parental altruism therefore shapes behavior but not the social objective: the government values each child independently of the weight κ placed on that child by their parents. Ellery, Tamer, and Weinzierl (2026) show that applying the original position to individuals rather than families likewise implies social weights on children that do not depend on their own parents' altruism, and they analyze the conflict with ex post Pareto efficiency that arises when parental altruism is heterogeneous.

expression on the right-hand side—uses equations (2), (19), and (21).

The optimal tax schedule may be characterized as follows:

Proposition 3 (Optimal Redistribution with Parental Investments). *For any given education schedule $E(\omega_0)$, the optimal marginal tax rate at each earnings level z satisfies*

$$\frac{\tau(z)}{1-\tau(z)} = \frac{1-G(z,A^*)}{(\varepsilon+\varepsilon_p)\,\alpha(z)}, \tag{25}$$

where ε is the direct earnings elasticity with respect to $1-\tau$ through children's supply responses, and ε_p is the indirect earnings elasticity with respect to $1-\tau$ through parents' education responses, defined in equation (24). The sum $\varepsilon+\varepsilon_p$ represents the macro elasticity of earnings accounting for parental education investments. As above, $\alpha(z) \equiv zh(z)/(1-H(z))$ denotes the local Pareto parameter, and $G(z,A^*)$ denotes the average social marginal welfare weight above earnings level z, measured at average preferences.

Proof. Equation (25) follows from the optimality condition $dM_\tau + dB_\tau + dB_\tau^p + dW_\tau = 0$, using the characterizations provided in equations (8)-(10) and (23). □

The optimal tax rule takes the same general form as in Proposition 1, but specified in terms of the macro elasticity of earnings, $\varepsilon+\varepsilon_p$, rather than the micro elasticity. The macro elasticity accounts for both the direct supply effect of taxes (ε) and the indirect effect operating through parental education investments (ε_p). Parental investment responses increase the earnings elasticity and therefore reduce optimal marginal tax rates, all else equal. This additional effect further clarifies why our general argument for predistribution differs from the arguments in the literature on education subsidies. Reducing income taxes on children to preserve parental education incentives resembles the efficiency rationale for education subsidies in Bovenberg and Jacobs (2005) and Stantcheva (2017). In our model, this mechanism operates through the tax system rather than the education schedule which, as we shall see below, is still governed by the productivity of public education and its potential for equity gains.

Let us turn to the optimal education schedule. We consider a small reform which increases public education provision by dE_g on the skill interval $(\omega_0, \omega_0+d\omega_0)$. The reform has the same conceptual effects as in the baseline model, but the characterizations need modification to account for parental education investments. Specifically, the impact of public education on skill includes both a direct effect and an indirect effect from parental responses. Using equation (22), we have

$$\frac{d\omega}{dE_g} = \Omega_g' + \Omega_p' \frac{dE_p}{dE_g} = \Omega_g' - \frac{\Omega_{pg}''\Omega_p'}{\Omega_{pp}''}, \tag{26}$$

where the second term captures the indirect effect from changes in parental investments. This term will

be negative if $\Omega''_{pg} < 0$ (crowding-out) and positive if $\Omega''_{pg} > 0$ (crowding-in). We note that equation (26) assumes that parents are at an interior solution. When parents are at a corner solution ($E_p = 0$)—i.e., when they fully free-ride on public education—there are no remaining private investments to crowd out and $d\omega/dE_g$ reduces to Ω'_g as in the baseline model. This case may arise when there is high substitutability between E_p and E_g and parents underinvest in the laissez-faire equilibrium relative to the social optimum, making it optimal for the government to crowd out parents entirely. Our simulations below show that this case is plausible when welfare weights at the bottom are above κ and when marginal tax rates are high, as they depress optimal parental investments while increasing optimal public investments due to fiscal externalities.

The three welfare effects of the perturbation to the education schedule can be characterized as follows:

$$dM_E^p = -\lambda \cdot p \cdot dE_g \cdot \int_A f_0(\omega_0|A)\, d\omega_0 dF_A(A) \tag{27}$$

$$dB_E^p = \lambda \cdot \left(\Omega'_g - \frac{\Omega''_{pg}\Omega'_p}{\Omega''_{pp}}\right) \cdot dE_g \cdot \int_A \tau(z)\frac{dz}{d\omega} \cdot f_0(\omega_0|A)\, d\omega_0 dF_A(A) \tag{28}$$

$$dW_E^p = \lambda \cdot g(\omega_0, A^*) \cdot \frac{du}{d\omega} \cdot \left(\Omega'_g - \frac{\Omega''_{pg}\Omega'_p}{\Omega''_{pp}}\right) \cdot dE_g \cdot \int_A f_0(\omega_0|A)\, d\omega_0 dF_A(A). \tag{29}$$

These expressions are similar to those derived in the baseline model, except for the term related to Ω''_{pg}. This term captures the potential crowding-out (or crowding-in) effects of public education provision. With these derivations, we may characterize the optimal education schedule:

Proposition 4 (Optimal Predistribution with Parental Investments). *For any given tax schedule $T(z)$, the optimal education provision at each level of background skill ω_0 satisfies*

$$\left(\Omega'_g - \frac{\Omega''_{pg}\Omega'_p}{\Omega''_{pp}}\right) \cdot \left[g(\omega_0, A^*)\frac{du}{d\omega} + \mathbb{E}\left[\tau(z)\frac{dz}{d\omega} \mid \omega_0\right]\right] = p, \tag{30}$$

where the first term in parentheses is the effect of education provision on skill, net of any crowding-out effects, as characterized in equation (26). As above, $\frac{du}{d\omega} = A^*(1-\tau(z))^{1+\varepsilon}/(1+\varepsilon)$ denotes the marginal willingness to pay for higher skill, and $g(\omega_0, A^*)$ denotes the social marginal welfare weight at background skill ω_0, measured at average preferences A^*.

Proof. At the social optimum, we have $dM_E^p + dB_E^p + dW_E^p = 0$. Inserting equations (27)-(29) and rearranging terms gives equation (30). □

The optimal education rule ensures that the social marginal benefit of education provision equals the social marginal cost. The only difference relative to the baseline model (Proposition 2) lies in the first term in parentheses, which now accounts for the potential crowding-out or crowding-in effects of government

provision.[21] Although this term is a complicated function of the derivatives of the Ω-function, it relates to an empirical reduced-form parameter: the elasticity of earnings with respect to government education spending. Using equations (2) and (26), we have

$$\zeta \equiv \frac{dz/z}{dE_g/E_g} = \left(\Omega'_g - \frac{\Omega''_{pg}\Omega'_p}{\Omega''_{pp}}\right) \cdot \frac{E_g}{\omega}. \tag{31}$$

Using this relationship, the education rule in equation (30) is, in a reduced-form sense, identical to the baseline rule in equation (15). The reason is that parental education responses matter only through their implications for the total earnings elasticity with respect to government provision, ζ. Empirically, this elasticity can be inferred from estimates provided in the literature on the impact of childhood education interventions on adult earnings. Any crowd-out effects from parental responses will be subsumed in reduced-form estimates of earnings effects in adulthood. As noted above, ζ is heterogeneous across earnings levels. In our simulation analysis, we calibrate $\Omega(.)$ to match empirical estimates of the average education elasticity $\mathbb{E}[\zeta]$ in the initial equilibrium.

To conclude, Propositions 3-4 show that parental investment responses can be captured by reinterpreting the reduced-form elasticities that govern optimal policy. Optimal redistribution depends on a macro elasticity of earnings with respect to taxes ($\varepsilon + \varepsilon_p$), whereas optimal predistribution depends on an elasticity of earnings with respect to government education spending that includes offsetting effects from crowding-out (ζ). However, while the optimal policy rules are unchanged in a reduced-form sense, the chosen policy parameters are not unchanged. The reduced-form elasticities are endogenous variables, and the policy rules are functions of their values *at the optimum*. For this reason, as emphasized by Kleven (2021), optimal policy analyses are not true sufficient statistics approaches. Even if expressed in terms of elasticities, they require global knowledge of supply and demand functions.

3.2 Allowing for Imperfect Tagging

The baseline model assumes that education provision $E(\omega_0)$ can be targeted based on exogenous family-background type ω_0. This may exaggerate the power of the predistribution instrument relative to taxes and transfers. In reality, family background is not perfectly observable and targeting may have to be

[21] Note that, when parents are at an interior solution and parental and government education investments are perfect substitutes, then Ω is a function of $E_p + E_g$. In this case, we have $\Omega'_p = \Omega'_g$ and $\Omega''_{pg} = \Omega''_{pp}$ such that the term $\Omega'_g - \Omega''_{pg}\Omega'_p/\Omega''_{pp}$ on the left-hand side of equation (30) equals zero. This corresponds to a scenario with full crowd-out, in which case government education provision (predistribution) is fully unproductive. Empirical evidence on the effects of education interventions and other predistribution policies is inconsistent with full crowd-out. In fact, some interventions have been found to crowd in parental investments (e.g., Gelber and Isen 2013; Attanasio *et al.* 2020). Even with full crowd-out, for $E_g = 0$ to be optimal, however, the government must prefer this allocation to the best possible allocation with $E_g > 0$ and $E_p = 0$. This is not necessarily satisfied: the optimal $E_g > 0$ with $E_p = 0$ may yield higher social welfare than $E_g = 0$ with privately optimal $E_p > 0$, particularly when welfare weights at the bottom are large and marginal tax rates are strongly positive.

based on choice variables. While imperfect tagging can be modeled in different ways, a natural scenario to consider is one where predistribution is based on parental income z_0 rather than parental type ω_0. To separate the role of imperfect tagging from the role of parental education investments, we introduce imperfect tagging in the baseline model without parental investments.

The utility function of both parents and children is given by equation (1), but we allow for different parameters in the two generations. Denoting parents by subscript zero, their earnings choices are characterized by $z_0 = A_0\omega_0 \left(1 - \tau_0 \left(z_0\right)\right)^{\varepsilon_0}$. This condition implies a mapping between parental skill and parental earnings, which we denote by $\omega_0 = \omega_0 \left(z_0, A_0\right)$.

Parents transmit labor market skill but not preferences to their children. The government provides education to children based on parental income, which depends on a combination of parental skill and preferences. Hence, skill production in the children's generation can be written as

$$\omega = \Omega \left(\omega_0, E \left(z_0\right)\right) = \Omega \left(\omega_0 \left(z_0, A_0\right), E \left(z_0\right)\right) \equiv \widehat{\Omega} \left(z_0, A_0\right), \tag{32}$$

where we use the aforementioned link between parental skill ω_0 and parental income z_0. The link depends on parental preferences A_0: if education provision is targeted to children from low-income families, this may reflect either low parental skill or low parental preferences for working. The fact that income-targeting partly captures parental preferences is the reason why tagging is imperfect in this model.

As above, we consider a government maximizing social welfare in the children's generation, taking policies in the parents' generation as given. The social welfare function may be specified in terms of parental income z_0 and parental preferences A_0, i.e.

$$W = \int_{A_0} \int_{z_0} \Psi \left[u \left(A^* \widehat{\Omega} \left(z_0, A_0\right)\right)\right] dF_{z_0} \left(z_0 | A_0\right) dF_{A_0} \left(A_0\right). \tag{33}$$

Social welfare depends on *average* preferences for working in the children's generation (A^*) and *individual* preferences for working in the parents' generation (A_0). The former reflects that the government applies the responsibility principle to preference heterogeneity, while the latter reflects that the level of background skill ω_0 affected by income-based education provision $E \left(z_0\right)$ depends on actual parental preferences.

The final piece of the government's problem is the budget constraint, which may be written as

$$\begin{aligned} \int_A \int_{A_0} \int_{z_0} T \left(z \left(A \widehat{\Omega} \left(z_0, A_0\right)\right)\right) dF_{z_0} \left(z_0 | A_0\right) dF_{A_0} \left(A_0\right) dF_A \left(A\right) \\ - p \int_{A_0} \int_{z_0} E \left(z_0\right) dF_{z_0} \left(z_0 | A_0\right) dF_{A_0} \left(A_0\right) \geq R, \end{aligned} \tag{34}$$

where we assume independence in the preferences for working across generations. The government

maximizes social welfare (33) subject to incentive compatibility constraints (2), skill production (32), and the government budget constraint (34).

The imperfect-tagging model does not change the optimal tax rule in equation (11), but it changes the optimal education rule. To characterize the optimal policy, suppose the government increases education provision by dE on the parental-earnings interval $(z_0, z_0 + dz_0)$. As in the baseline model, such a perturbation has a mechanical revenue effect, a behavioral revenue effect, and a direct welfare effect. These effects may be characterized as follows:

$$dM_E = -\lambda \cdot p \cdot dE \cdot dz_0 \cdot \int_{A_0} f_{z_0}(z_0|A_0)\, dF_{A_0}(A_0) \tag{35}$$

$$dB_E = \lambda \cdot dE \cdot dz_0 \cdot \mathbb{E}\left[\tau(z)\frac{dz}{d\omega}\Omega'_E \mid z_0\right] \cdot \int_{A_0} f_{z_0}(z_0|A_0)\, dF_{A_0}(A_0) \tag{36}$$

$$dW_E = \lambda \cdot dE \cdot dz_0 \cdot \mathbb{E}\left[g(z_0, A_0, A^*)\frac{du}{d\omega}\Omega'_E \mid z_0\right] \cdot \int_{A_0} f_{z_0}(z_0|A_0)\, dF_{A_0}(A_0), \tag{37}$$

where we define $g(z_0, A_0, A^*) \equiv \Psi'\left[u\left(A^*\widehat{\Omega}(z_0, A_0)\right)\right]/\lambda$ as the social marginal welfare weight on individuals with parental background variables z_0, A_0. Using these expressions, we obtain the following result:

Proposition 5 (Optimal Predistribution with Imperfect Tagging). *For any given tax schedule $T(z)$, the optimal education provision at each level of parental income z_0 satisfies*

$$\mathbb{E}\left[\Omega'_E \cdot \left(g(z_0, A_0, A^*)\frac{du}{d\omega} + \tau(z)\frac{dz}{d\omega}\right) \mid z_0\right] = p, \tag{38}$$

where $du/d\omega$ denotes the marginal willingness to pay for higher skill, and $g(z_0, A_0, A^*)$ denotes the social marginal welfare weight on children with parental income z_0 and parental preferences A_0, conditional on average preferences for working in the children's generation A^*.

Proof. The result follows from the optimality condition $dM_E + dB_E + dW_E = 0$, using equations (35)-(37) and rearranging terms. □

The optimal education rule is similar to the baseline rule in Proposition 2. Formally, imperfect tagging changes the result in two ways: (*i*) the social marginal benefit of government provision is an average at parental income z_0, and (*ii*) the social marginal welfare weight is defined over parental income and preferences z_0, A_0 rather than parental skill ω_0. All else equal, the latter effect implies less progressivity in education provision because parental income is only a noisy signal of deservingness (background skill ω_0) due to preference heterogeneity.

3.3 General Dynamic Framework with Imperfect Tagging

The preceding sections simplify dynamics by considering models with only two generations—parents and children—where the government maximizes social welfare in the children's generation, taking policies in the parents' generation as given. It is possible to generalize the analysis to account for multiple generations and a government that sets tax and education schedules in each generation to maximize dynamic social welfare. This section lays out the general framework, with the characterization of optimal policies presented in Appendix D.

Our focus is on intergenerational dynamics rather than lifecycle dynamics within a generation. We consider a setting in which non-overlapping generations live for one period.[22] Each generation t consists of a continuum of mass 1 of workers with heterogeneous skill ω_t and preferences for working A_t. The joint distribution of skill and preferences is given by the density function $f_t\left(\omega_t|A_t\right) f_{A_t}\left(A_t\right)$. The distribution of skill ω_t is predetermined within a generation, but it may change from generation to generation in response to education provision.

The utility of a worker of type (A_t, ω_t) is given by

$$u_t = z_t - T_t\left(z_t\right) - \frac{A_t\omega_t}{1+1/\varepsilon}\left(\frac{z_t}{A_t\omega_t}\right)^{1+1/\varepsilon}, \tag{39}$$

the maximization of which yields earnings supply:

$$z_t = A_t\omega_t\left(1-\tau_t\left(z_t\right)\right)^{\varepsilon}. \tag{40}$$

The earnings capacity of each worker—defined as earnings at $\tau_t = 0$—is given by $A_t\omega_t$. Indirect utility is written as $u_t\left(A_t\omega_t\right)$.

As above, the key departure from a Mirrleesian environment is to endogenize skill with respect to a pre-market instrument such as education provision. This instrument affects the intergenerational transmission of skill. Specifically, skill production takes the following form:

$$\omega_t = \Omega\left(\omega_{t-1}, E_{t-1}\left(z_{t-1}\right)\right), \tag{41}$$

where $\Omega\left(.\right)$ is non-decreasing in both arguments. The skill of an individual in generation t depends on the previous generation through two mechanisms. The first is a draw ω_{t-1} from the skill distribution of the previous generation. This captures the effect of family background: ω_{t-1} can be interpreted as the

[22] More precisely, each generation of individuals makes active labor market decisions in only one period, but the individuals are alive during the previous period—i.e., when their parents make labor market decisions and when they may benefit from pre-market education provision.

skill level of one's parents. The second mechanism is the effect of government education provision for individuals of that family background, $E_{t-1}(z_{t-1})$. These investments are made during the working life of generation $t-1$ and are therefore pre-market from the perspective of generation t. Note that education provision depends on (endogenous) parental earnings, thereby allowing for imperfect tagging as in the previous section.

Inserting lagged values of skill into equation (41), we may rewrite intergenerational skill production as follows:

$$\omega_t = \widehat{\Omega}\left(\omega_0, E_0(z_0), ..., E_{t-1}(z_{t-1})\right). \tag{42}$$

Hence, the skill of a worker in generation t depends on the baseline skill of their dynasty (ω_0) and on the entire history of education investments in their ancestors and themselves ($E_0, ..., E_{t-1}$). This implies that, if the government changes education investments in a given generation, the effect cascades through future generations due to intergenerational skill transmission. The strength of such cascading depends on the magnitude of $\Omega'_{\omega_{t-1}}$ in equation (41). If $\Omega'_{\omega_{t-1}}$ is small (large), intergenerational cascading fades quickly (slowly).

The government has an equal-opportunity objective: it respects inequality driven by preferences A_t, but not inequality driven by predetermined skill ω_t. Social welfare is specified as

$$W = \sum_{t=0}^{T} \int_{A_t} \int_{\omega_t} \delta_t \Psi\left[u_t\left(A_t^* \omega_t\right)\right] dF_{\omega_t}\left(\omega_t | A_t\right) dF_{A_t}\left(A_t\right), \tag{43}$$

where δ_t is a Pareto weight on generation t, and $A_t^* \equiv \int_{A_t} A_t dF_{A_t}(A_t)$ denotes average preferences for working in generation t.[23] When setting tax and education schedules, the government must satisfy a dynamic budget constraint:

$$\sum_{t=0}^{T} \int_{A_t} \int_{\omega_t} \beta^t \left[T_t\left(z_t\left(A_t \omega_t\right)\right) - p E_t\left(z_t\left(A_t \omega_t\right)\right)\right] dF_{\omega_t}\left(\omega_t | A_t\right) dF_{A_t}\left(A_t\right) \geq R, \tag{44}$$

where β is a discount factor. Here we assume that the price of education investments is constant across generations, but this is trivial to generalize.

The government sets generation-specific tax and education schedules $\{T_t(z_t), E_t(z_t)\}_{t=0}^{T}$ to maximize social welfare (43) subject to incentive compatibility (40), skill production (41), and the government budget constraint (44). We solve this optimization problem in Appendix D and provide dynamic policy rules similar in spirit to the static rules presented above. Allowing for general dynamics does not change the fundamental insights of the model, but it increases the empirical and numerical complexity by

[23] Alternatively, we could specify social welfare as a function of average preferences across all generations: $A^* \equiv \sum_{t=0}^{T} \int_{A_t} A_t dF_{A_t}(A_t) / (T+1)$. This specification choice does not affect the results in any substantive way.

an order of magnitude. Solving the general model numerically is complicated—especially with many generations—because the tax and education schedules rely on endogenous entities that interact both within and across generations. The optimal policy in any given generation depends on potential effects many generations down the line—effects that are difficult to evaluate and are unlikely to be internalized by policymakers. Hence, the simpler models presented above have greater empirical and policy relevance.

4 Quantifying Optimal Predistribution and Redistribution

In this section we present numerical simulations that illustrate the quantitative importance of predistribution relative to traditional redistribution. Using our baseline model from Section 2, we tailor the calibrations to match a prominent predistribution policy: public expenditure on K-12 education. We first consider a simplified version of the model that abstracts from preference heterogeneity (in Sections 4.1-4.2) and then extend the simulations to allow for heterogeneity in preferences for working (in Sections 4.3-4.4), and for parental investments in education (in Section 4.5). The full details of the numerical simulations are provided in Appendix E.

4.1 Calibration Details

The simulations require four functional forms. First, normalizing the preference parameter A to one, the utility function (1) may be written as

$$u\left(z;\omega_0\right)=z-T\left(z\right)-\frac{\widehat{\Omega}\left(\omega_0\right)}{1+1/\varepsilon}\left(\frac{z}{\widehat{\Omega}\left(\omega_0\right)}\right)^{1+1/\varepsilon}, \tag{45}$$

where earnings supply equals $z=\widehat{\Omega}\left(\omega_0\right)\left(1-\tau\left(z\right)\right)^{\varepsilon}$ by equations (2)-(3).

Second, the technology of skill production (3) is assumed to be additively separable in family-background skill ω_0 and government education investments $E\left(\omega_0\right)$. Specifically, we have

$$\omega=\widehat{\Omega}\left(\omega_0\right)=\Omega\left(\omega_0,E\left(\omega_0\right)\right)=\omega_0+B\cdot E\left(\omega_0\right)^{\eta}. \tag{46}$$

The parameter B governs the aggregate productivity of government investments, while η governs their returns to scale. In the baseline model, we abstract from parental education investments, but these will be incorporated in an extension below.

Third, we assume that background skill ω_0 is log-normally distributed with parameters $\left(\mu_{\omega_0},\sigma_{\omega_0}\right)$.[24]

[24]While log-normal distributions induce thinner tails than the ones observed empirically (Diamond 1998; Saez 2001), we refrain from appending a Pareto tail because, as we detail in Appendix E, the log-normal assumption allows us to exactly decompose the distribution of $A\omega_0$ into log-normal marginal distributions of A and ω_0, facilitating the analysis of the case with

Finally, the social welfare function takes a standard CRRA form, $\Psi(u) = \frac{u^{1-\rho}}{1-\rho}$, where the coefficient of relative risk aversion ρ governs inequality aversion. Our baseline calibration assumes $\rho = 1$, corresponding to the logarithmic case.

We set parameters based on external evidence or calibration as follows. The earnings elasticity ε with respect to the net-of-tax rate is set equal to 0.5 at baseline, broadly consistent with the empirical literature on intensive margin responses (Chetty *et al.* 2011; Saez, Slemrod, and Giertz 2012; Kleven *et al.* 2025). We investigate comparative statics with respect to ε. The returns-to-scale parameter η is set equal to 0.75, the midpoint of structural estimates surveyed in Browning, Hansen, and Heckman (1999).[25] We jointly calibrate the skill-distribution parameters $(\mu_{\omega_0}, \sigma_{\omega_0})$ and the productivity of government education spending B to match three empirical moments: the mean and standard deviation of the current U.S. labor income distribution (obtained from the five most recent waves of the Panel Study of Income Dynamics) and an empirical estimate of the average elasticity of earnings with respect to government education spending, $\mathbb{E}[\zeta]$.

Specifically, we define the empirical moment $\mathbb{E}[\zeta]$ as the average elasticity of adult earnings with respect to government spending on K-12 education. Table 1 details our construction of ζ from the empirical literature. Jackson and Mackevicius (2024) survey twenty-six quasi-experiments on government K-12 spending in the U.S. to estimate its average per-dollar causal impact on test scores. Chetty, Friedman, and Rockoff (2014) study as-good-as-random assignments of students to teachers to estimate effects of teachers on both childhood test scores and later-life adult earnings, the ratio of which yields an adult earnings impact per childhood test score impact. Combining these two values yields our preferred estimate for ζ of 0.216, local to the below-average parent-income sample of Chetty, Friedman, and Rockoff (2014). Given the decreasing returns to scale $\eta = 0.75$, our calibration implies a population average $\mathbb{E}[\zeta]$ equal to 0.196. We consider the implications of alternative values of $\mathbb{E}[\zeta]$ in comparative statics analyses.

The calibration is implemented based on an approximation of the current policy environment in the U.S. We assume that every type ω_0 receives the same amount of public education spending, $E(\omega_0) = \overline{E}$, equal to the 2024 annual average K-12 spending per pupil of $20,668 (U.S. Census Bureau 2026). We empirically estimate the current tax system $T(z)$ using the microfiles of Piketty, Saez, and Zucman (2018), as in Gaubert *et al.* (2025). This procedure yields $(B, \mu_{\omega_0}, \sigma_{\omega_0}) = (2.78, 3.95, 1.09)$.

preference heterogeneity.

[25] This structural literature follows the Ben-Porath tradition where earnings are a function of educational dollars, educational time (sometimes called on-the-job training), and initial human capital. Browning, Hansen, and Heckman (1999) report a range of 0.5 to 0.99 for the returns to scale on educational dollars plus educational time. More recent papers report estimates of similar orders of magnitude: Manuelli and Seshadri (2014) estimate returns to scale of 0.89 for flow educational dollars, while Attanasio *et al.* (2020) estimate returns to scale of 0.52-0.59 for material investments but negligible returns to time.

4.2 Results Without Preference Heterogeneity

Having calibrated the model, we solve numerically for the policy vector $\{T(z), E(\omega_0)\}_{\forall z,\omega_0}$ that satisfies the optimal policy rules in equations (11) and (15). Details on the solution algorithm are provided in Appendix E.

Figure 2 plots optimal profiles of policy parameters by percentile of background skill ω_0: marginal and average tax rates in Panels A-B and education spending per child in Panel C. The figure shows optimal tax rates both when the planner can predistribute (our model in red) and when the planner cannot (standard Mirrleesian model in black). The education schedule is highly progressive: optimal spending is monotonically declining in background skill, falling from nearly \$50,000 per child at the bottom to almost zero at the top.[26] Optimal spending is about six times larger in the bottom percentiles than at the median. The equalizing effect of education provision relieves pressure on the tax-transfer system to equalize incomes. As a result, the optimal tax system becomes less progressive. With predistribution, marginal tax rates are everywhere lower, average tax rates are flatter, and the demogrant (not shown) is much smaller (\$8,553 vs \$23,541).[27]

Figure 3 illustrates the income-equalizing effects of predistribution and redistribution at the optimal policy. It shows density distributions of background skill ω_0, realized skill ω, and after-tax income $z - T(z)$ in our predistribution model (Panel A) and in the standard Mirrleesian model (Panel B). The difference between the densities of ω_0 and ω represents the effects of predistribution, while the difference between the densities of ω and $z - T(z)$ represents the effects of redistribution. The effects of predistribution are substantial. Progressive education spending boosts realized skill (i.e., potential earnings) at the bottom of the distribution, raising the lower bound from roughly zero to \$30,000. Progressive taxes further raise the lower bound (but by less) and reduce incomes at the top. By contrast, when traditional redistribution is the only option, taxes and transfers are left to do more income equalization on their own. They cannot compensate fully, however, and the distribution of after-tax income in Panel B is therefore more dispersed than in Panel A.

Table 2 examines the effects of predistributive and redistributive policies on different measures of inequality. It reports inequality in background skill ω_0, realized skill ω, and after-tax income $z - T(z)$ across a range of different indicators. As above, predistribution corresponds to the difference between ω_0 and ω, while redistribution corresponds to the difference between ω and $z - T(z)$. Considering the Gini

[26] Assuming 13 years of childhood education and 30 years of work, Figure 2C scales education spending by 30/13 to ensure that spending per child in our static, lifetime framework is comparable to real-world numbers (see Appendix E).

[27] Because the distribution of types is bounded, labor supply responses are at the intensive margin, and the bottom type has positive ω_0 and, therefore, works, we replicate the standard result from optimal tax theory that the bottom and top marginal tax rates equal zero. Replicating the U-shaped pattern of Diamond (1998) and Saez (2001) requires an unbounded distribution with a thick tail (such as Pareto) and a bottom type who does not work (see, e.g., Piketty and Saez 2013).

coefficient, we find that predistribution reduces the coefficient from 0.56 to 0.48 and that redistribution further reduces it to 0.30. As a result, predistribution is responsible for 31.5% of the aggregate reduction in this inequality measure. Relative to the Mirrleesian benchmark, optimal policy reduces the Gini coefficient by an extra 12%, from 0.56 to 0.30 instead of only to 0.34.

The role of predistribution is larger for inequality indicators that emphasize changes at the bottom of the distribution. As shown in the table, the share of predistribution equals 44%, 39%, 56%, 72%, and 80% when considering the bottom 10% share, the bottom 25% share, the Palma ratio, the P90/P10 ratio, and the P50/P10 ratio, respectively.[28] The improvement in inequality relative to the Mirrleesian benchmark is also larger for these indicators. This pattern matches those shown in Figures 2-3: the progressivity of the education profile compresses the distribution of ω primarily by boosting the bottom types, with relatively small effects at the top of the distribution. Conversely, allowing for predistribution has a modest impact on inequality when focusing on indicators that emphasize the top. For example, when focusing on the top 1% share, the share of predistribution falls to 24% and the improvement over Mirrlees is only 6%.

To gain a better understanding of the tradeoff between predistribution and redistribution, we analyze the comparative statics with respect to the different parameters that govern this tradeoff. To economize on exposition, we focus on the Gini coefficient. Figure 4 presents comparative statics with respect to the tax elasticity ε, which summarizes the efficiency costs of redistribution.[29] The figure shows that, when ε goes to zero, the planner implements perfect equality in after-tax incomes using taxes and transfers, as in the standard Mirrleesian model.[30] As ε increases, the efficiency costs of taxation constrain the optimal level of redistribution. However, the availability of a predistributive instrument relaxes this constraint, allowing the planner to substitute from tax-transfer policy towards education provision as ε increases. At $\varepsilon = 1$, more than half of the policy-induced reduction in the Gini coefficient is achieved by predistribution.

Figure 5 provides comparative statics with respect to the population-average education elasticity $\mathbb{E}[\zeta]$, which represents the productivity of public education. The analysis is implemented by recalibrating B in equation (46), holding the returns-to-scale parameter η constant, to match different values of $\mathbb{E}[\zeta]$ at the baseline policy environment. The relative importance of predistribution responds strongly to this parameter. As $\mathbb{E}[\zeta]$ falls to 0.1 or less, the role of predistribution disappears and the optimum converges to the Mirrleesian benchmark. Conversely, as $\mathbb{E}[\zeta]$ increases above its baseline value of 0.196,

[28] The Palma ratio is defined as the ratio of the top 10% share to the bottom 40% share (Palma 2011).

[29] Changing ε has two effects: it affects the efficiency cost of redistribution—the channel we want to highlight in Figure 4—but it also affects the welfare benefit of predistribution by changing the marginal willingness to pay for higher skill, $\frac{du}{d\omega} = \frac{A(1-\tau(z))^{1+\varepsilon}}{1+\varepsilon}$. To isolate the redistribution-cost channel in the comparative statics, for each value of ε, we recalibrate A such that $\frac{du}{d\omega}$ is constant at the baseline tax system.

[30] We note that predistribution remains relevant even when redistribution is unrestricted ($\varepsilon = 0$): with no residual inequality, the planner provides a uniform level of education, $E(\omega_0) = \overline{E}$, for efficiency reasons. This mechanically decreases the Gini coefficient.

predistribution increases strongly and the optimal Gini coefficient falls substantially. At $\mathbb{E}[\zeta] = 0.3$, the income-equalizing effect of government policy is driven almost entirely by predistribution.

The baseline analysis assumes that the marginal productivity of public education, $\Omega'_E = B\eta E^{\eta-1}$, is independent of family-background type ω_0, conditional on spending E. However, public education investments may vary in their productivity across families. To assess the implications of such heterogeneity, we specify $B(\omega_0) = BC_\xi \exp\{\xi F_0(\omega_0)\}$ where, for a given value of ξ, the scalar C_ξ is chosen such that $\mathbb{E}[B(\omega_0)] = B$. In this parameterization, ξ represents the spread in $B(\omega_0)$ between the top and bottom types: $\xi = \log B(\overline{\omega_0}) - \log B(\underline{\omega_0})$. Hence, ξ governs the slope of education productivity with respect to type, with $\xi > 0$ ($\xi < 0$) representing the case where education is more productive for children with high (low) family background. Figure 6 presents comparative statics with respect to ξ. When education productivity increases with family background ($\xi > 0$), the optimal policy involves less predistribution and more redistribution. Intuitively, it is more efficient to invest in children at the top in order to increase their incomes as adults and exploit the greater tax base via higher tax rates. Conversely, if education investments are more productive at the bottom of the family-background distribution, predistribution is larger and inequality is smaller at the social optimum. These results demonstrate the importance of estimating not only the average education elasticity $\mathbb{E}[\zeta]$, but also how the elasticity varies with family background.

Figures 7 and 8 reproduce comparative statics for the education parameters—the average education elasticity $\mathbb{E}[\zeta]$ and the slope of education productivity ξ—using inequality indicators other than the Gini coefficient. Figure 7 considers the P90/P10 ratio. Consistent with the findings in Table 2, the share of predistribution in inequality reduction is much larger for the P90/P10 ratio than for the Gini. The comparative statics are qualitatively similar, but the predistribution share is larger in any given calibration. Figure 8 turns to the intergenerational elasticity of earnings (IGE), a widely used indicator in the social mobility literature (see, e.g., Chetty *et al.* 2014).[31] The IGE at the optimal policy is smaller in cases where predistribution plays a greater role, namely when education is more productive overall ($\mathbb{E}[\zeta]$ high) or when the productivity gains are concentrated at the bottom (ξ low). Because the laissez-faire equilibrium of our model has an IGE of one, Figure 8 implies that the effects of predistribution on the IGE may be very large.

Finally, Appendix Figure A.2 presents comparative statics with respect to the inequality-aversion parameter ρ. As inequality aversion increases, the planner implements a lower Gini coefficient through a combination of pre- and redistribution. In the limit of Rawlsian social preferences ($\rho \to \infty$), the planner

[31] In the context of our model, we define the IGE in terms of earnings potential ω, i.e. IGE $= d\log\omega / d\log\omega_0$. Given our utility specification, and assuming that income taxes are the same in the two generations, this is equivalent to $d\log z / d\log z_0$. Because our model abstracts from random income shocks (such that there is perfect transmission of background skill ω_0), IGEs and rank-rank correlations in the absence of government intervention equal one.

reduces the Gini coefficient in after-tax incomes to 0.18, with almost 60% of the reduction driven by predistribution. This finding is consistent with Figures 2-3, in which we saw that predistribution is the preferred instrument for boosting the incomes of those from the lowest family backgrounds. At the opposite extreme, when preferences for redistribution vanish ($\rho \to 0$), the government provides a uniform amount of education, $E(\omega_0) = \overline{E}$, for efficiency reasons, funded by a regressive tax system. The resulting inequality in after-tax incomes is slightly larger than inequality in family-background skill.

4.3 Results with Preference Heterogeneity

The baseline simulations isolate the effect of one of our model's innovations: introducing a predistribution instrument into an otherwise Mirrleesian framework. We now extend the simulations to allow for heterogeneity in preferences for working, A.

Individuals are characterized by a two-dimensional vector of background types (A, ω_0), where we simplify by assuming that A and ω_0 are independently distributed. We denote earnings capacity (before education investments) by $\widetilde{\omega} = A\omega_0$. Our objective is to study, for a fixed distribution of $\widetilde{\omega}$, how optimal policy changes as a larger share of the variation in $\widetilde{\omega}$ is explained by variation in A. Our simulations in the previous section correspond to the limit case where none of the variation in $\widetilde{\omega}$ is explained by A. Another limit case is where ω_0 is constant across individuals, so that all of the variation in $\widetilde{\omega}$ is explained by A. Leveraging the independence between A and ω_0, we operationalize the intermediate cases of this decomposition with the R^2 of a (weighted) linear regression of $\widetilde{\omega}$ on A. When A is homogeneous, we have $R^2 = 0$. When ω_0 is homogeneous, we have $R^2 = 1$. Intermediate cases have $R^2 \in (0, 1)$, providing an interpretable measure of the relative importance of preferences in driving earnings capacity.[32]

The challenge is to compute different pairs of marginal distributions of A and ω_0 that yield the same distribution of $\widetilde{\omega} = A \cdot \omega_0$, but different values of the relative importance of preferences as measured by the R^2 statistic described above. We fix the distribution of $\widetilde{\omega}$ using the calibration of the baseline model without preference heterogeneity: a log-normal distribution with parameters $(\mu_{\widetilde{\omega}}, \sigma_{\widetilde{\omega}}) = (3.95, 1.09)$.[33] Leveraging the independence of A and ω_0, we assume that each of the marginal distributions is also log-normal with parameters (μ_A, σ_A) and $(\mu_{\omega_0}, \sigma_{\omega_0})$. Based on known properties of log-normal distributions, this gives $\mu_{\widetilde{\omega}} = \mu_A + \mu_{\omega_0}$ and $\sigma_{\widetilde{\omega}} = \sqrt{\sigma_A^2 + \sigma_{\omega_0}^2}$. The R^2 of the regression of $\widetilde{\omega}$ against A can be written as a function of the marginal log-normal parameters: $R^2 = \frac{\exp\left(\sigma_A^2\right)-1}{\exp\left(\sigma_A^2+\sigma_{\omega_0}^2\right)-1}$ (see Appendix E). This yields an identification result: given $(\mu_{\widetilde{\omega}}, \sigma_{\widetilde{\omega}})$, the values of $\mathbb{E}[\omega_0]$, $\mathbb{E}[A]$, and R^2 jointly determine (μ_A, σ_A) and

[32] Standard variance decompositions do not work in this case because we are concerned with the product, not the sum, of A and ω_0.

[33] More generally, we note that the calibration above remains valid for this exercise as the decomposition between A and ω_0 affects optimal policy through the definition of welfare weights, but not the labor supply decisions in the absence of policy.

$(\mu_{\omega_0}, \sigma_{\omega_0})$. Hence, we can back out marginal log-normal distributions of A and ω_0 that induce the exact distribution of $\widetilde{\omega} = A \cdot \omega_0$ and have constant means $\mathbb{E}[\omega_0]$ and $\mathbb{E}[A]$, but whose variances vary to generate the different values of the $R^2 \in (0, 1)$.

Given the set of marginal distributions, we numerically solve for the optimal policy at each value of R^2 (see Appendix E for details). The Gini decomposition for total inequality is presented in Panel A of Figure 9. Consistent with our theoretical analysis, the figure shows that, as the importance of preference heterogeneity increases, the planner equalizes incomes by less through either redistribution or predistribution. The reason is that, with greater heterogeneity in A, less of inequality is welfare-relevant and so the profile of welfare weights flattens, consistent with the arguments in Lockwood and Weinzierl (2015) and Berg, Håvarstein, and Stubhaug (2025). In the limit where $R^2 = 1$, the planner implements a regressive tax system to finance uniform education provision, $E(\omega_0) = \overline{E}$, motivated solely by efficiency concerns.[34]

However, this aggregate pattern masks an important aspect of predistribution and its interaction with preference heterogeneity. Panel B of Figure 9 reproduces Panel A, but focuses on the welfare-relevant portion of inequality driven by family-background ω_0. It shows Gini coefficients in ω_0, ω, and $\mathbb{E}[z - T(z)|\omega_0]$. Mechanically, the level of welfare-relevant inequality decreases with R^2. More interestingly, as R^2 increases, a greater share of the policy-induced reduction in welfare-relevant inequality is achieved by predistribution. The reason is the tagging argument discussed above: as preference heterogeneity increases, earnings become less informative about opportunity and therefore a worse tag.

To reinforce these points, Panel C of Figure 9 plots a measure of the progressivity of education spending against the relative importance of preferences as measured by the R^2. For each R^2, we compute the average elasticity of education spending $E(\omega_0)$ with respect to background skill ω_0, $-d \log E(\omega_0) / d \log \omega_0$, which coincides with the object $-\mathbb{C}(\log E(\omega_0), \log \omega_0) / \mathbb{V}(\log \omega_0)$.[35] This measure, which resembles standard scalar measures of the progressivity of a tax system (Musgrave and Thin 1948), provides insights into the compression effects of education per unit of remaining opportunity inequality. The figure shows that the progressivity of education increases with the importance of preference heterogeneity. The planner makes education spending more progressive, as the education instrument targets the welfare-relevant component of inequality. This increase in education progressivity is not visible in Panel A because a larger variance in preferences A mechanically lowers the variance in background skill ω_0 such that education has

[34] Note that constant education does not affect the Gini coefficient before taxes and transfers. This differs from Figure A.2 where, in the limit without inequality aversion ($\rho = 0$), constant education spending does reduce the Gini. In that analysis, , because constant education spending is added to heterogeneous background types ω_0, inequality falls as the constant is *proportionally* larger for the low types. In Figure 9 with $R^2 = 1$, however, constant education spending is added to homogeneous background types ω_0, implying that the skill increase is proportional and therefore distributionally neutral. See Appendix F for a formal argument.

[35] We define this elasticity in absolute value (as $E(\omega_0)$ is non-increasing in ω_0 at the optimum).

a smaller impact on aggregate inequality, but becomes visible in Panel B as it abstracts from the variance of A.

4.4 Empirical Estimate of Preference Heterogeneity

The preceding section highlights the share of earnings variation explained by preferences A relative to family background ω_0 as an important statistic for policy design. To separate the two sources of earnings variation empirically, we implement a method in the tradition of Roemer and Trannoy (2016), Hufe *et al.* (2017), and Hufe, Kanbur, and Peichl (2022). The method is based on the R^2 of two sequential regressions: a regression of wage income on family-background variables (opportunity regression), and a regression of wage income on both family background and labor supply variables (opportunity+choice regression). The role of preferences is based on the difference between the two R^2-estimates. The sequential procedure implies that any covariance between family background and labor supply is assigned to the effects of family background.[36] To put it differently, the role of preferences is based on the correlation between earnings and labor supply choices above and beyond what can be explained by family-background variables.

Using data from the 1979 cohort of the National Longitudinal Survey of Youth (NLSY79), we classify covariates into vectors of opportunity O and choice C and conduct the following sequential procedure. First, we regress wage earnings on the opportunity vector O and denote by R^2_O the adjusted R^2 of that regression. We then regress wage earnings on both the opportunity O and choice C vectors and denote by R^2_{O+C} the adjusted R^2 of that regression. With these measures, we build two benchmarks for the relative importance of preferences. First, we consider the metric $R^2_{O+C} - R^2_O$, which we consider a lower bound on the share of earnings variation explained by choices as it assigns all the unexplained variation to opportunity. Second, we consider the metric $\left(R^2_{O+C} - R^2_O\right) / R^2_{O+C}$, which we call the "proportional benchmark" as it assigns the unexplained variation to opportunity and choice in proportion to their explained shares. In both benchmarks, our sequential procedure assigns all the covariance between O and C to opportunity given standard omitted variable bias mechanics.

When implementing this exercise, the researcher must take a stand on which variables belong to the vectors O and C, which remains contentious in the literature (Rawls 1971; Nozick 1974; Roemer and Trannoy 2016). We roughly adopt an age-18 cutoff: characteristics determined before age 18 belong in O while characteristics determined after age 18 belong in C. Our preferred specification defines O to include parent income, parent employment, parent age, parent education, sex, race, year of birth, country

[36] For example, if family background and labor supply variables were *perfectly* correlated, our sequential procedure would assign none of the earnings variation to preference heterogeneity, as in the Coleman Report's decomposition of student achievement into family background and school components (Coleman *et al.* 1966, cf. Hoxby 2016).

of birth, and the Armed Forces Qualification Test score (an aptitude measure). We define C to include the child's own annual hours worked, four-year-lagged annual hours worked, own educational attainment, family choices like age at first birth and marital status, and crime and health behaviors like the number of delinquent acts and the number of cigarettes per day. All continuous variables are included as quartics.

Table 3 displays the results from opportunity-only regressions in Panel A and opportunity+choice regressions in Panel B. Panel A begins by regressing the child's own wage earnings in 2013 (when children were aged 48-56) on their parents' income, defined as quartics in family income in each year 1978-1981 plus additional variables for parent employment, poverty, and age. We find that 9.5% of the variation in children's earnings in adulthood ($R^2_O = 0.095$) is explained by differences in parental income, similar to the 8.0% of the variation in child earnings rank explained by parent income rank found in administrative tax returns (Chetty *et al.* 2014). In our preferred specification, opportunity covariates explain roughly 23% ($R^2_O = 0.229$) of the variation in the child's own wage earnings. The sequential addition of opportunity covariates in our baseline set incrementally increases the adjusted R^2. However, numerous other opportunity variables available in the NLSY79 have relatively little impact, including childhood family size, religion, newspaper subscription, library card holding, and psychological test scores for locus of control and self-esteem. Panel B shows that when adding hours worked—the first choice covariate—the adjusted R^2 rises substantially to 0.371. Adding the remaining choice variables yields $R^2_{O+C} = 0.414$: observed opportunity and choice variables together explain 41% of the variance in 2013 earnings.

In the context of our metrics, the values $R^2_O = 0.229$ and $R^2_{O+C} = 0.414$ imply a lower bound on the share of earnings variation explained by choices equal to 18.5% and a proportional benchmark share of 44.7%. Returning to Figure 9, we see that x-axis values of 18.5% and 44.7% imply a substantial role for predistribution. This is especially clear when focusing on the welfare-relevant portion of inequality in Panel B. Specifically, in the proportional benchmark of $R^2 = 44.7\%$, the share of predistribution is almost 50%, compared to 31.5% in the simulations without preference heterogeneity.

4.5 Results with Parental Education Investments

The final set of results examines the effect of allowing for parental investments in children's education, following the extended model in Section 3.1. To simplify, these simulations abstract from preference heterogeneity. The full details of the analysis are presented in Appendix E.

We assume that the skill production function takes a CES form:

$$\omega = \Omega\left(\omega_0, E_p\left(\omega_0\right), E_g\left(\omega_0\right)\right) = \omega_0 + B\left[E_p\left(\omega_0\right)^\delta + E_g\left(\omega_0\right)^\delta\right]^{\eta/\delta}. \tag{47}$$

In this specification, $\phi = 1/(1-\delta)$ is the elasticity of substitution between public and parental investments. When $\delta = \eta$, the skill production function reduces to $\omega_0 + B \cdot E_p(\omega_0)^\eta + B \cdot E_g(\omega_0)^\eta$ in which case $\Omega''_{pg} = 0$. In this case, there is neither crowd-in nor crowd-out of parental investments: $dE_p/dE_g = 0$ when $\Omega''_{pg} = 0$ by equation (22). In our baseline calibration with $\eta = 0.75$, the case of $\Omega''_{pg} = 0$ corresponds to an elasticity of substitution of $\phi = 4$. For all $\delta < \eta$ ($\phi < 4$), skill production exhibits crowd-in, whereby public investments increase parental investments ($\Omega''_{pg} > 0$). Conversely, for all $\delta > \eta$ ($\phi > 4$), skill production exhibits crowd-out, whereby public investments decrease parental investments ($\Omega''_{pg} < 0$). In the limit where $\delta \to 1$ ($\phi \to \infty$), there is perfect substitutability between public and parental education investments.

We recalibrate the extended model to match the same empirical moments as in the baseline model, accounting for the effects of parental investments on the earnings distribution and on the earnings elasticity with respect to government education. In this recalibration, we assume that the weight placed by parents on their children is $\kappa = 1$ (i.e., parents fully internalize their children's welfare in equation 18) and that $\delta = \eta = 0.75$ (so that skill production satisfies $\Omega''_{pg} = 0$), but investigate the comparative statics with respect to both parameters. Specifically, we conduct the following comparative-static exercises to provide insights into the role of parental investments for optimal policy. First, we solve for the optimal policy at different parental weights κ, assuming that κ is homogeneous across parents. Second, we allow for heterogeneity in κ, focusing on the empirically realistic case where parental education investments are regressive (i.e., where κ is increasing in family-background skill). We specify $\kappa(\omega_0) = C_\nu \cdot \omega_0^\nu$, where $\nu \geq 0$ is the elasticity of the parental weight with respect to family-background skill, and where the scalar C_ν is chosen (for each ν) to ensure that $\mathbb{E}[\kappa(\omega_0)] = 1$. We then solve for the optimal policy at different values of the elasticity ν. Third, we fix $\kappa = 1$ for all families and analyze the comparative statics with respect to the elasticity of substitution ϕ. This exercise shows how the presence of crowd-in or crowd-out affects the optimal mix between redistribution and predistribution.

Figure 10 shows our results. Panel A presents comparative statics with respect to κ. When $\kappa = 0$, $E_p(\omega_0) = 0$ for all ω_0 and the equilibrium corresponds to the baseline case without parental investments.[37] As κ increases, parents increase their educational investments which reduces inequality, as shown in Panel A. The reason is that, by the first-order condition (20), when κ is homogeneous and absent income effects on education investments, the only source of variation in $E_p(\omega_0)$ is variation in marginal tax rates. The presence of progressive marginal tax rates implies that parental investments decline in family-background skill, reducing the pre-market dispersion of skill, all else equal. Government policy further reduces inequality using pre- and redistribution, the composition of which is not very sensitive to the level of κ.

[37] To clarify, the Gini coefficients at $\kappa = 0$ do not coincide exactly with the baseline estimates in Table 2 because the simulations are based on recalibrated values of B, μ_{ω_0}, and σ_{ω_0}.

Because an inequality-reducing profile of parental investments is unrealistic, Panel B relaxes this Panel A feature by allowing parental education weights $\kappa(\omega_0)$ to be increasing in family-background skill. As described above, we implement this by varying the elasticity of the parental weight, ν. The case where $\nu = 0$ replicates Panel A with $\kappa = 1$. As ν increases, parental education investments eventually turn regressive, making the distribution of skills after parental investments more unequal than the distribution of background skill. The government offsets this inequality using both pre- and redistribution, and the composition between the two is highly robust to the size of ν. The general lesson from these analyses is that the importance of predistribution is not particularly sensitive to either the level or the profile of private education investments.

The preceding panels assume $\delta = \eta$ and, therefore, abstract from crowd-in or crowd-out effects. To understand the implications of such effects, Panel C varies the elasticity of substitution between government and parental education, $\phi = 1/(1-\delta)$. Unsurprisingly, when skill production exhibits crowd-in ($\phi < 4$), optimal predistribution is stronger. In this case, government education provision leads to larger parental investments, increasing the social value of predistribution. On the other hand, and perhaps more surprisingly, crowd-out of parental investments ($\phi > 4$) has little effect on the optimal mix of pre- and redistribution and on the Gini coefficient at the optimum. This holds even in the limit of $\phi \to \infty$, where parental and public investments are perfect substitutes. What explains this?

Part of the explanation can be found in Panel D. There we plot the share of public education spending in total education spending. In the baseline without crowd-in or crowd-out ($\phi = 4$), the government is responsible for more than 80% of all education spending. The government's desired education spending is much larger than parents' desired spending for two reasons. First, the presence of positive marginal tax rates discourages parents from investing in their children's education (see equation 20), while it encourages the government to invest more due to the fiscal externality. Second, social marginal welfare weights $g(\omega_0)$ are greater than one at the bottom of the distribution, so that the government values the marginal utility of children more than their parents do at $\kappa = 1$. The two forces together yield $E_g(\omega_0) \gg E_p(\omega_0)$. As a consequence, even as crowd-out becomes stronger, the government remains incentivized to invest in education in order to match its much larger target. As shown in Panel D, this results in an increase in the share of public education spending. Furthermore, while crowd-out reduces the social marginal return to public investments, non-negativity constraints on $E_p(\omega_0)$ imply that, as parental investments converge toward zero, the incentive compatibility constraint (20) eventually becomes irrelevant to the government. Put simply, when parents invest close to zero, there are virtually no private investments to crowd out and the government can freely implement its desired level of education. In the limit, the government behaves as in the baseline case with inactive parents: parents are at a corner

solution for $E_p(\omega_0)$ and the government is the exclusive provider of education.

The bottom line of this analysis is that crowd-out effects have limited implications for the optimal degree of predistribution, which remains substantial even in the extreme case of perfect substitutability between public and private investments.[38]

5 Conclusion

Governments around the world use taxes and transfers to redistribute income and predistributive policies such as education provision to promote equality of opportunity. Yet the canonical Mirrleesian framework in economics abstracts from pre-market policy instruments and social preferences for equality of opportunity. This paper develops a normative framework that incorporates predistribution and equality-of-opportunity concerns, using it to derive rules for optimal education provision and optimal taxes and transfers in terms of empirically estimable sufficient statistics.

Our framework implies that optimal redistribution follows a generalized Diamond-Saez rule, while optimal education provision follows a generalized Samuelson rule. Predistribution plays a larger role relative to redistribution when taxes are more distortionary; when predistributive spending has larger effects on earnings, particularly among children from disadvantaged families; and when earnings differences are driven by preferences for work rather than unequal opportunities. Calibrating our model to U.S. data, we find that optimal policy achieves a larger reduction in inequality than in the Mirrleesian framework, but that it relies much less on progressive taxes and transfers to do so.

Our framework points to several directions for future theoretical and empirical research. First, our model could be extended to evaluate policies that address disparities frequently discussed in policy debates about opportunity, such as racial, gender, and geographic differences. In particular, the model could be adapted to analyze the optimal role of investments and policies targeted to particular groups or places. One could also extend the framework to consider policies that change how skills translate into earnings, such as collective bargaining regulations or trade restrictions, complementing our focus on investments that develop skills. These extensions would broaden the set of instruments that can be

[38] Two remarks follow. First, the corner solution in the limit with $\phi \to \infty$ does not exactly coincide with the baseline case with inactive parents because, for a few types at the top of the distribution, the government does not value the provision of public education more than parents do (see Panel C of Figure 2). For those types, the planner prefers to reach the corner solution of $E_g(\omega_0) = 0$ and delegate all the educational investments to those parents. Quantitatively, this nuance is irrelevant: in the simulations with $\phi \to \infty$, this happens only at the top 5% of the background skill distribution, for which parental investments are low enough so that, in the aggregate, public education represents 99.9% of total education investments. Second, the result that public provision does not vanish under perfect substitution has precedent in the literature on private contributions to public goods and charitable donations. While it is well known that perfect substitution may lead to perfect crowd-out and, therefore, no incentives for public provision when individuals are marginal contributors (Bergstrom, Blume, and Varian 1986), Roberts (1984) shows that public provision is still optimal with free-riding individuals when the public valuation of the good is above the private one and, therefore, individuals are no longer marginal contributors.

evaluated jointly with taxes and transfers under an equality-of-opportunity objective.

Second, beyond normative guidance, our model could be applied to understand existing policy differences from a political economy perspective. For example, as discussed in the introduction, European countries have less income inequality than the United States, despite redistributing less, because their pre-tax incomes are more equally distributed. The greater persistence of earnings across generations in the U.S. (Corak 2013) suggests one potential explanation for its greater reliance on redistribution: holding other factors fixed, our model implies a greater optimal role for redistributive taxation when earnings differences arise from differences in family background. The lower level of intergenerational mobility in the U.S. today may arise in part from the legacy of excluding some subgroups (e.g., Black Americans) from education and other opportunities (Althoff and Reichardt 2024). Hence, the greater reliance on redistribution in the U.S. today could be due in part to different policies shaping equality of opportunity in the past. Evaluating such hypotheses and, more generally, extending the model to incorporate political and historical forces could shed further light on the origins of current policy institutions.

Finally, our framework provides guidance for future empirical research by highlighting the parameters that matter for policy design. One key parameter is the share of earnings variation attributable to preferences versus opportunity. Existing measures of intergenerational income mobility capture the association between parents' and children's incomes, focusing on one dimension of opportunity. Future work could broaden this approach by eliciting how the public distinguishes opportunities from choices through surveys, and then measuring the share of earnings variation explained by the circumstances identified as sources of unequal opportunity. Another key input is how the effects of education and other predistributive policies on earnings vary across the family-background distribution. Identifying more effective ways to improve disadvantaged children's skills would be especially valuable because such policies can both narrow pre-market inequality and reduce the need for distortionary redistribution. Further empirical research on these questions can inform not only which specific programs merit more investment, but how governments should combine investments in creating opportunity with taxes and transfers to maximize social welfare.

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TABLE 1: ESTIMATE OF ζ: THE ELASTICITY OF ADULT EARNINGS WITH RESPECT TO K-12 EDUCATION SPENDING

A. Causal Impact of Government K-12 Education Spending on Test Scores (Jackson and Mackevicius 2024)	
Causal impact of four years of $1,000 (in 2018$) of government K-12 education spending on standard deviations of test scores	0.0316 (0.0056)
B. Percent Earnings Increase per Standard Deviation Impact on Test Scores (Chetty, Friedman, and Rockoff 2014)	
Percent earnings increase per standard deviation increase in test scores	12.7% (3.3%)
Inputs to calculation based on teacher value-added impacts (12.7% = $350 / $21,256 / 0.13):	
Causal impact of one year of one std. dev. higher teacher value-added on age-28 wage earnings	$350 ($92)
Age-28 mean earnings within the sample population (precisely estimated)	$21,256
Causal impact of one year of one std. dev. higher teacher value-added on test scores (precisely estimated)	0.13
C. Other Inputs	
Per-pupil K-12 annual expenditure in 2024 (Census)	$20,668
CPI-U inflation 2018-2024 (BLS)	24.9%
D. Implied Estimate of Education Elasticity ζ	
Estimate of the elasticity ζ of adult earnings with respect to government K-12 education spending, equal to:	0.216
Percent effect on earnings dz/z (= 0.0316 x 0.127)	0.4%
Divided by: Percent dosage of government education spending dE/E (= 4 x $1,000 x 1.249 / (13 x 20,668))	1.9%
Implied population-average estimate of ζ, reflecting the CFR sample's below-average parent income (numerical)	0.196

Notes: Panel A reports a meta-analysis estimate of the impact of government K-12 spending on student test scores, based on twenty-six U.S. quasi-experiments (Jackson and Mackevicius 2024, Table 2 column 1). Panel B reports estimates of test-score impacts and adult earnings impacts of quasi-experimental variation in teacher exposure in a large U.S. school district (Chetty, Friedman, and Rockoff 2014, Table 3, column 1 and Section III.C). Panel C lists other inputs. Panel D uses Panels A-C to compute an estimate of the elasticity ζ of adult earnings with respect to government K-12 education spending. The computation's assumptions are that the Panel A studies are representative of the test score impacts of government education spending generally and that the Panel B ratio of the percentage adult earnings impact per standard deviation of test score impact applies to Panel A's test score impact. ζ is heterogeneous and declines with background skill. In our numerical simulation that yields $\zeta = 0.216$ on average for the bottom 89% of types (matching CFR's mean parent income), we obtain $\zeta = 0.196$ on average across all types.

TABLE 2: THE EFFECTS OF POLICY ON DIFFERENT INEQUALITY INDICATORS

	Predistribution Model				Mirrleesian Model	**Improvement over Mirrlees**
	ω_0	ω	$z - T(z)$	**Share $\omega_0 - \omega$**		
Gini	0.56	0.48	0.30	**31.5%**	0.34	**12.0%**
Bottom-Tail Emphasis:						
Bottom 10%	0.9%	3.0%	5.7%	**44.2%**	4.5%	**26.8%**
Bottom 25%	4.1%	8.3%	14.9%	**39.0%**	12.5%	**18.6%**
Palma Ratio	4.7	2.7	1.2	**55.5%**	1.4	**16.0%**
P90/P10	16.4	6.7	2.9	**72.2%**	3.7	**20.9%**
P50/P10	4.0	1.8	1.3	**80.2%**	1.5	**17.6%**
Top-Tail Emphasis:						
Top 1%	11.3%	10.4%	7.3%	**24.0%**	7.8%	**6.0%**
Top 10%	42.9%	39.4%	28.9%	**24.9%**	30.5%	**5.2%**
P90/P50	4.1	3.7	2.3	**23.5%**	2.4	**4.0%**

Notes: This table reports inequality in background skill ω_0, realized skill ω, and after-tax income $z - T(z)$ at the optimal policy in the baseline calibration. The difference between ω_0 and ω reflects predistribution, while the difference between ω and $z - T(z)$ reflects redistribution. The different rows consider different inequality indicators. The Palma ratio is defined as the ratio of the top-10% share to the bottom-40% share. The first four columns present results from our predistribution model, where the planner jointly optimizes tax-transfer policy and education provision. The column labeled "Share $\omega_0 - \omega$" shows the share of the total inequality reduction—from background skill to after-tax income—achieved through predistribution. That is, if $\Gamma(x)$ is the inequality indicator for the variable x, then "Share $\omega_0 - \omega$" is calculated as $(\Gamma(\omega_0) - \Gamma(\omega)) / (\Gamma(\omega_0) - \Gamma(z - T(z)))$. The table also reports inequality in after-tax income at the Mirrleesian benchmark (achieved through taxes and transfers alone), and the inequality improvement (in percentage terms) in our predistribution model relative to the Mirrleesian benchmark.

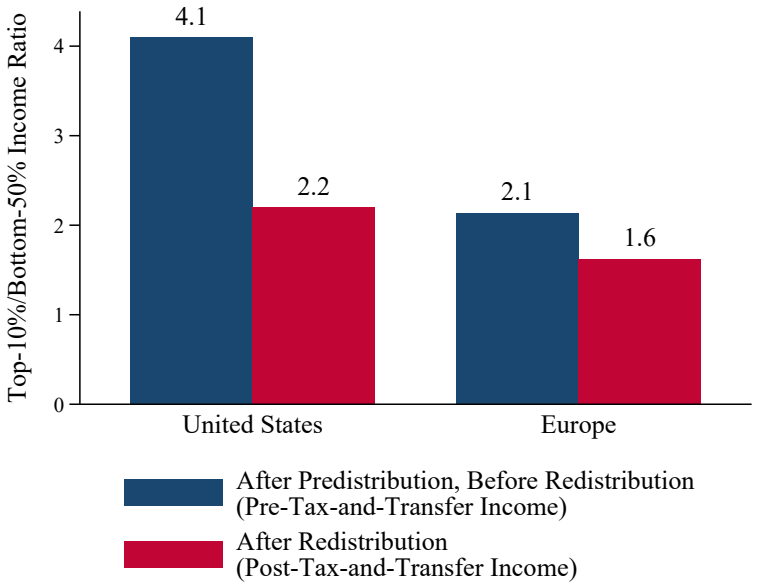
TABLE 3: DECOMPOSITION OF EARNINGS INTO OPPORTUNITY VS CHOICE

	Child's Own Adult Wage Earnings in 2013					
	(1)	(2)	(3)	(4)	(5)	(6)
A. Opportunity Covariates						
Parent income	X	X	X	X	X	X
Parent education		X	X	X	X	X
Demographics			X	X	X	X
AFQT score				X	X	X
Eight other childhood environment					X	X
Five other own endowment						X
R^2_O	0.095	0.129	0.176	0.229	0.234	0.240
B. Opportunity+Choice Covariates						
Opportunity covariates						
Parent income	X	X	X	X	X	X
Parent education	X	X	X	X	X	X
Demographics	X	X	X	X	X	X
AFQT score	X	X	X	X	X	
Choice covariates						
Hours worked	X	X	X	X	X	X
Lagged hours worked		X	X	X	X	X
Educational attainment			X	X	X	X
Family choices				X	X	X
Crime and health behaviors					X	X
R^2_O	0.229	0.229	0.229	0.229	0.229	0.176
R^2_{O+C}	0.371	0.384	0.404	0.414	0.414	0.404
$R^2_{O+C} - R^2_O$	14.2%	15.5%	17.5%	18.5%	18.5%	22.7%
$(R^2_{O+C} - R^2_O)$ / R^2_{O+C}	38.3%	40.3%	43.4%	44.7%	44.7%	56.3%

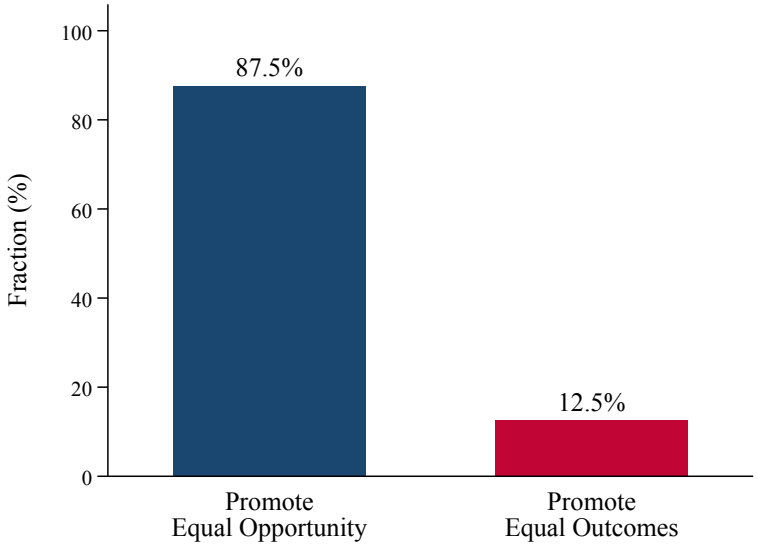
Notes: This table presents adjusted R^2 statistics from regressions of 2013 individual wage earnings in the 1979 National Longitudinal Survey of Youth (NLSY79). Each column of each panel represents a separate regression and includes indicators for missing values of all included covariates. All regressions are estimated on the same sample of 6,782 individuals. Parent income comprises quartics in family income in each year 1978-1981, mother's and father's years of birth, indicators for poverty in each year 1978-1981, and indicators for mother's and father's employment. Parent education comprises indicators for mother years of education and father years of education. Demographics comprise indicators for race, sex, birth year, and country of birth. AFQT score comprises a quartic in the 1980-measured and 2006-re-scored Armed Forces Qualification Test score; we obtain a nearly identical increase in the adjusted R^2 between Panel A columns 3 and 4 when restricting the sample to birth cohorts 1962-1964, which were 18 years old or younger in 1980. Eight other childhood environment variables comprise a quartic in 1979 family size and indicators for religion raised in, newspaper subscription at age 14, magazine subscription at age 14, library card at age 14, interactions of parent types living with the respondent at age 14, and approximately one hundred occupational categories each for mother's occupation and father's occupation. Five other own endowment variables comprise quartics in height, 1979 Rotter's Internal-External Locus of Control score, and 1980 Rosenberg Self-Esteem Scale score and indicators for years of education top-coded at 12 (when the typical respondent was 18 years old) and 1979 health limitations. Hours worked comprises a quartic in hours worked in 2013 and an indicator for being employed at the 2014 interview date. Lagged hours worked comprises a quartic in hours worked in 2009 and an indicator for being employed at the 2010 interview date. Educational attainment comprises indicators for non-top-coded years of education. Family choices comprise quartics in age at first birth and spouse hours per week and indicators for being married in 2014 and number of children in the household in 2014. Crime and health behaviors comprise quartics in the number of distinct types of delinquent acts and the number of cigarettes per day and an indicator for ever having been incarcerated.

FIGURE 1: MOTIVATING FACTS

A. Europe's Lower Inequality Is Entirely Explained by Lower Inequality After Predistribution and Before Redistribution



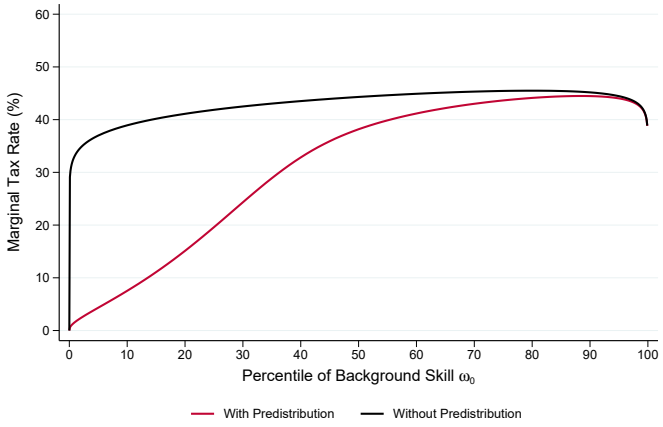
B. People Favor Promoting Equality of Opportunity



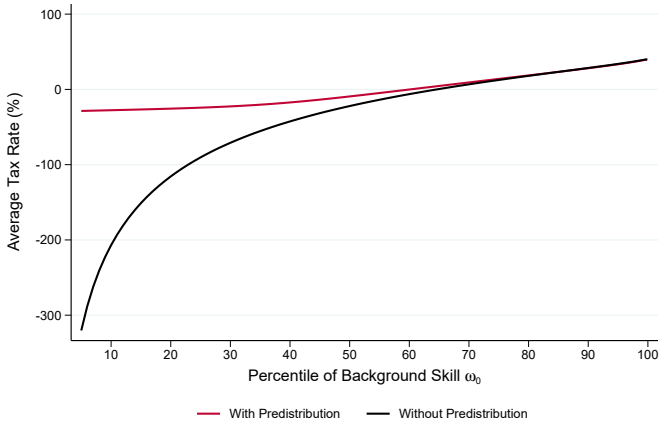
Notes: This figure presents motivating facts. Panel A uses data from Blanchet, Chancel, and Gethin (2022) to plot a summary measure of inequality—the ratio of top-10% income to bottom-50% income—in 2017, the most recent year available, for the United States and Europe (26 countries, weighted by population). For each region, the panel plots inequality measured at two points: before taxes and transfers ("After Predistribution and Before Redistribution") and after taxes and transfers ("After Redistribution"). Panel B reports preferences for promoting equal opportunity vs promoting equal outcomes based on the 1993 wave of the General Social Survey (GSS), the only wave that includes this particular question. The exact wording of the question is as follows: "Some people think America should promote equal opportunity for all, that is, allowing everyone to compete for jobs and wealth on a fair and even basis. Other people think America should promote equal outcomes, that is, insuring that everyone has a decent standard of living and that there are only small differences in wealth and income between the top and bottom in society. Which do you favor: promoting equal opportunity or promoting equal outcomes?" The answer options were "Promote equal opportunity", "Promote equal outcomes", or "Other/Don't know/No answer." Shares are computed using GSS person post-stratification weights. We omit the 1% that answered in the third category and scale up the first two categories to sum to 100%. See Appendix B for additional details, including GSS-reported beliefs on the determinants of why a person's life turns out well or poorly.

FIGURE 2: OPTIMAL POLICY PARAMETERS

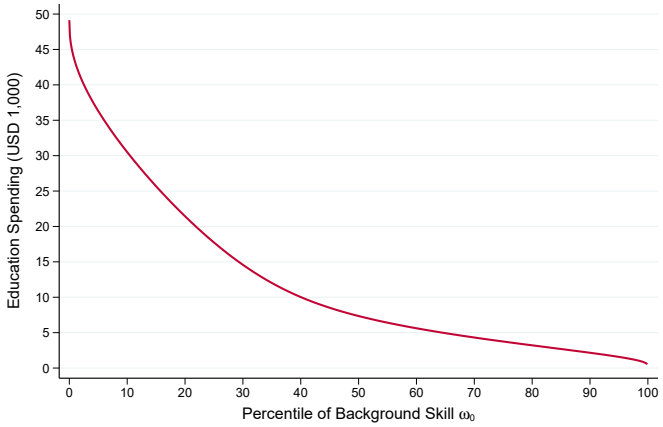
A. Redistribution
Marginal Tax Rate Profile



B. Redistribution
Average Tax Rate Profile



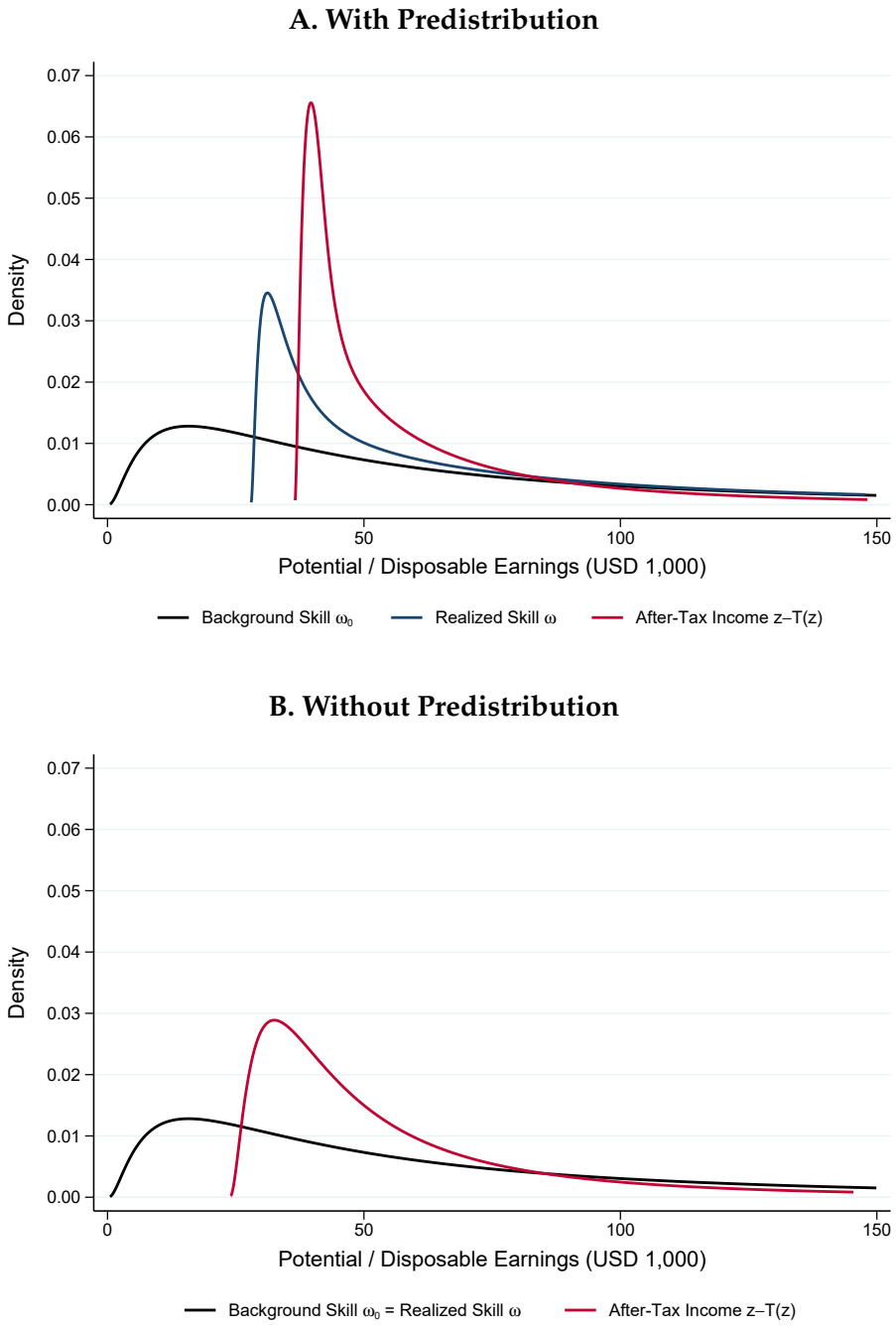
C. Predistribution
Education Profile



Notes: This figure plots the optimal policy functions by percentile of background skill ω_0. Panel A displays the profile of marginal tax rates. Panel B displays the profile of average tax rates, $T(z)/z$, truncating the x-axis at the 5th percentile for readability. In these panels, the red curves ("With Predistribution") depict optimal tax policy functions when the planner is also optimizing education provision, while the black curves ("Without Predistribution") depict optimal tax policy functions without any public education provision (Mirrleesian benchmark). Panel C displays the optimal profile of education spending, $E(\omega_0)$.

FIGURE 3: EFFECT OF OPTIMAL POLICY ON EARNINGS DISTRIBUTIONS

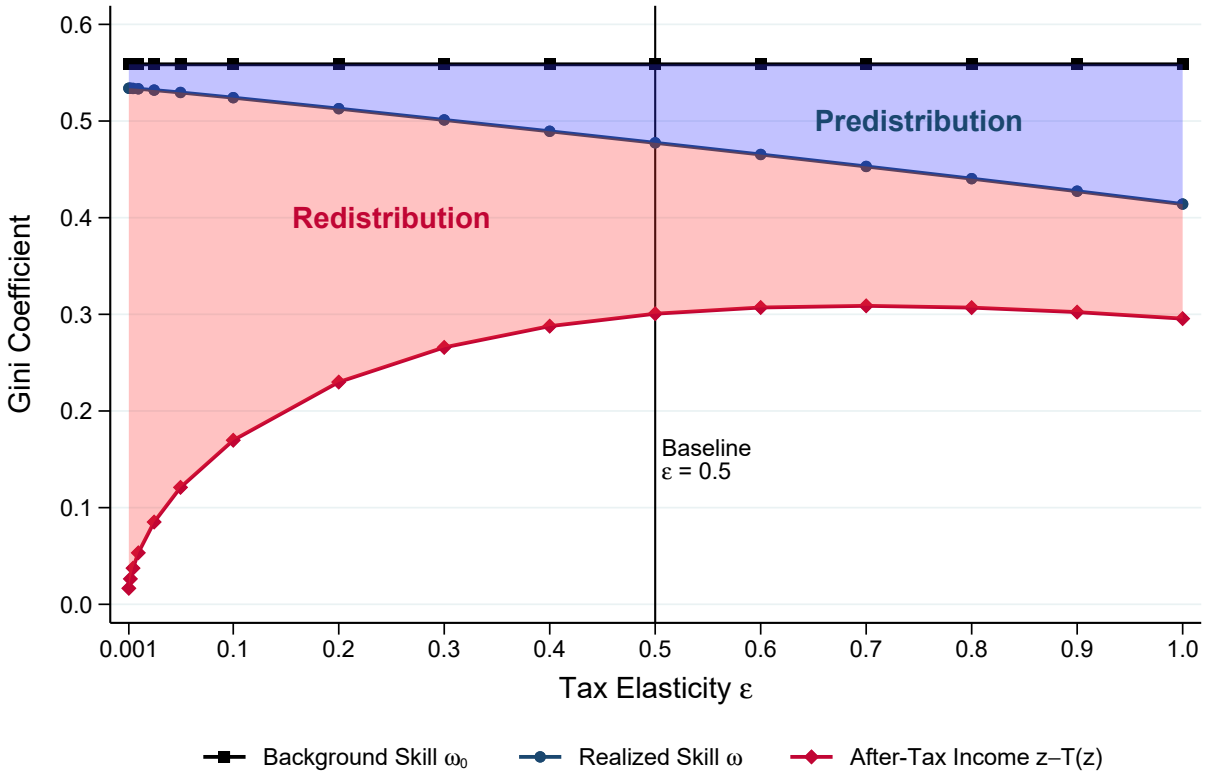
DENSITIES OF ω_0, ω, AND $z - T(z)$ AT THE OPTIMAL POLICY



Notes: This figure plots the densities of background skill ω_0, realized skill ω, and after-tax income $z - T(z)$ at the optimal policy. Panel A ("With Predistribution") considers the case where the planner jointly optimizes taxes and education provision. Panel B ("Without Predistribution") considers the case where the planner only optimizes taxes (Mirrleesian benchmark without public education provision), in which case the distributions of background skill ω_0 and realized skill ω are identical. In both panels, the x-axis is truncated at 150 for readability purposes.

FIGURE 4: EFFECT OF EARNINGS ELASTICITY WRT. $1-\tau$

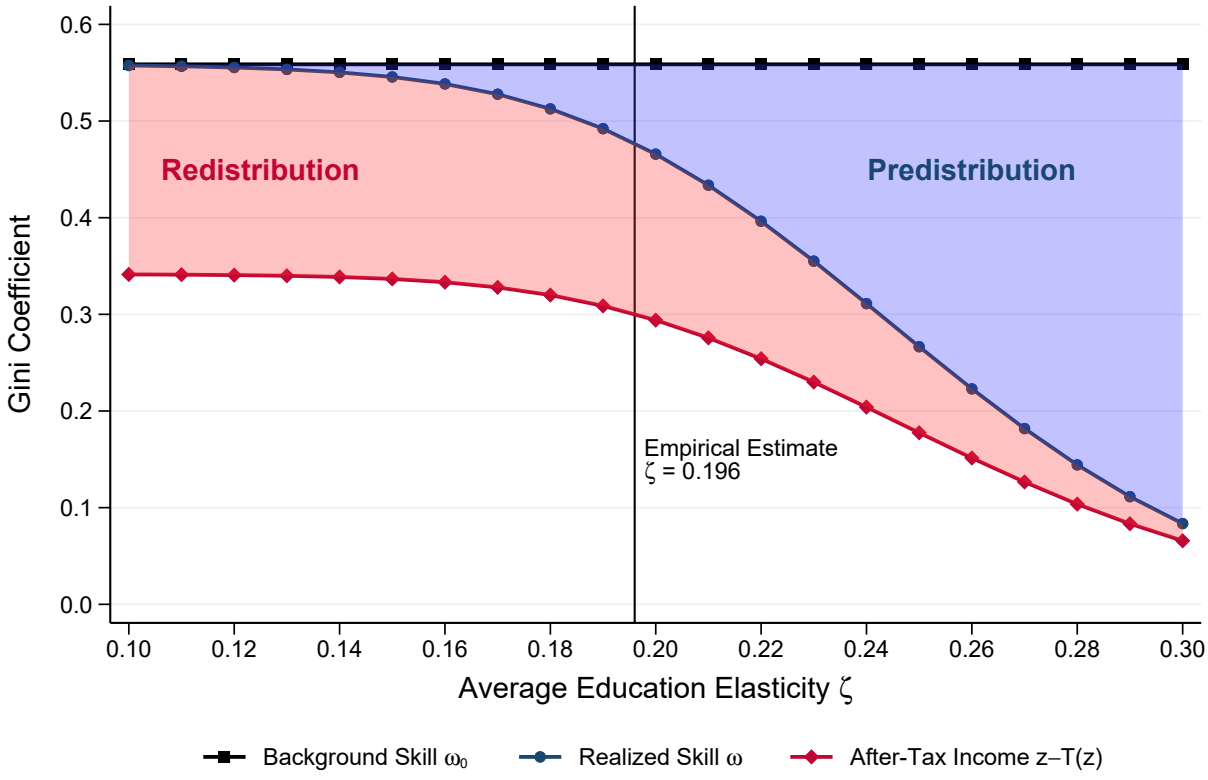
DECOMPOSITION OF THE GINI COEFFICIENT



Notes: This figure plots the Gini coefficient of background skill ω_0 (black squares), realized skill ω (blue circles), and after-tax income $z - T(z)$ (red diamonds) for different values of the tax elasticity ε. The difference between ω_0 and ω reflects predistribution, while the difference between ω and $z - T(z)$ reflects redistribution. When changing ε, the scalar parameter A is recalibrated to ensure that the marginal willingness to pay for higher skill, $du/d\omega = A(1-\tau(z))^{1+\varepsilon}/(1+\varepsilon)$, is held constant at the baseline tax system.

FIGURE 5: EFFECT OF EARNINGS ELASTICITY WRT. EDUCATION

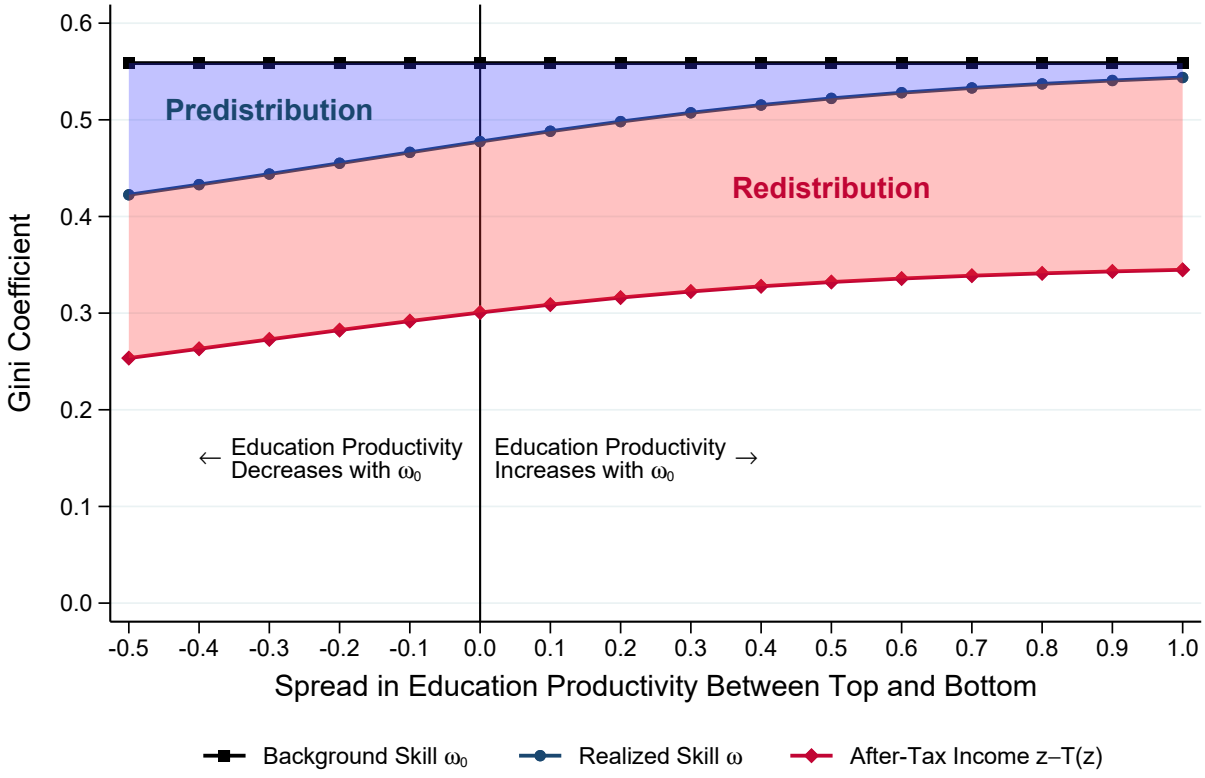
DECOMPOSITION OF THE GINI COEFFICIENT



Notes: This figure plots the Gini coefficient of background skill ω_0 (black squares), realized skill ω (blue circles), and after-tax income $z - T(z)$ (red diamonds) for different values of the average education elasticity $\mathbb{E}[\zeta]$. The difference between ω_0 and ω reflects predistribution, while the difference between ω and $z - T(z)$ reflects redistribution. The analysis is implemented by recalibrating B in equation (46), holding the returns-to-scale parameter η constant, to match different values of $\mathbb{E}[\zeta]$ at the baseline policy environment.

FIGURE 6: EFFECT OF HETEROGENEITY IN EDUCATION PRODUCTIVITY

DECOMPOSITION OF THE GINI COEFFICIENT

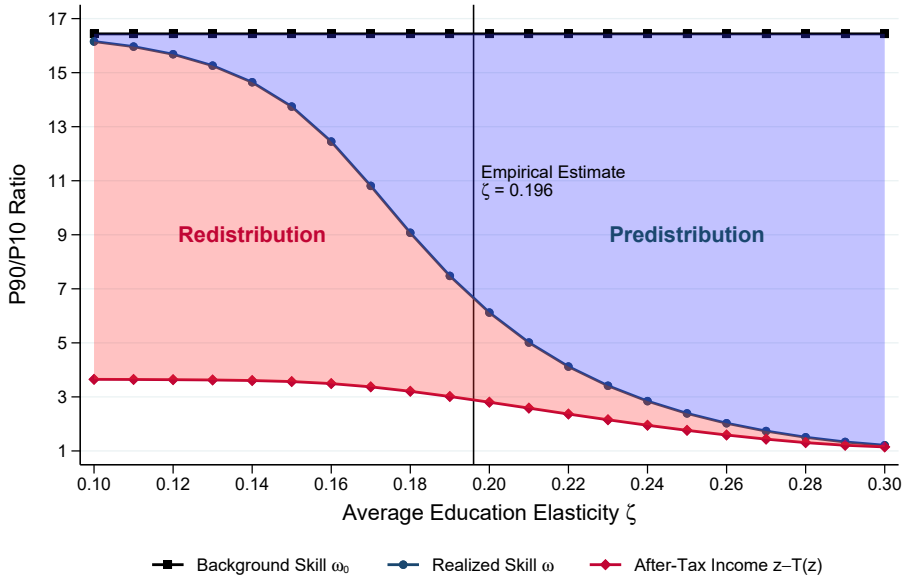


Notes: This figure plots the Gini coefficient of background skill ω_0 (black squares), realized skill ω (blue circles), and after-tax income $z - T(z)$ (red diamonds) for different degrees of heterogeneity in marginal education productivity. Specifically, we scale education productivity at ω_0 by $B(\omega_0) = BC_\xi \exp\{\xi F_0(\omega_0)\}$, where ξ represents the spread in $B(\omega_0)$ between the top and bottom of the family-background distribution: $\xi = \log B(\overline{\omega_0}) - \log B(\underline{\omega_0})$. Hence, ξ is a measure of the slope of the education productivity with respect to family background, with $\xi > 0$ ($\xi < 0$) representing the case where education is more productive for children with high (low) family background. The figure shows the effects of predistribution (difference between ω_0 and ω) and redistribution (difference between ω and $z - T(z)$) as we vary the extent and direction of heterogeneity measured by ξ.

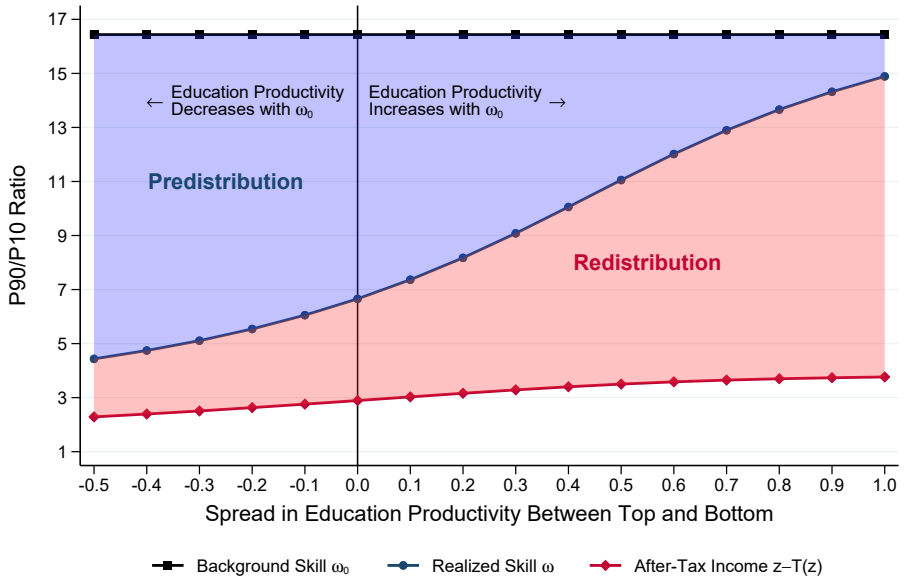
FIGURE 7: DECOMPOSITION OF THE P90/P10 RATIO

COMPARATIVE STATICS

A. Effect of Earnings Elasticity wrt. Education

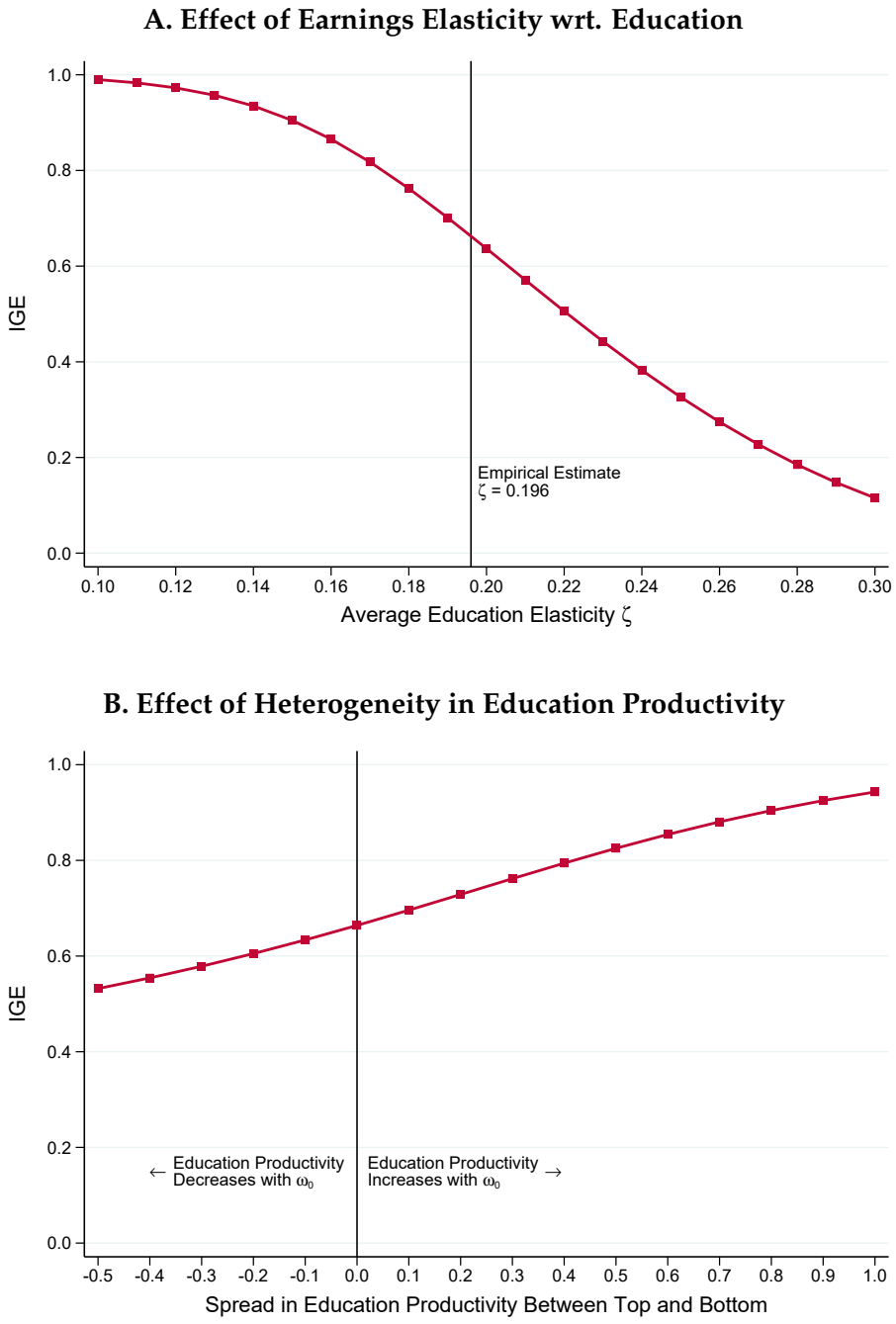


B. Effect of Heterogeneity in Education Productivity



Notes: This figure reproduces the comparative statics of Figures 5-6 using the P90/P10 ratio as the inequality indicator instead of the Gini coefficient. All other details of the simulations are the same as those specified in the notes of Figures 5-6 and in Appendix E.

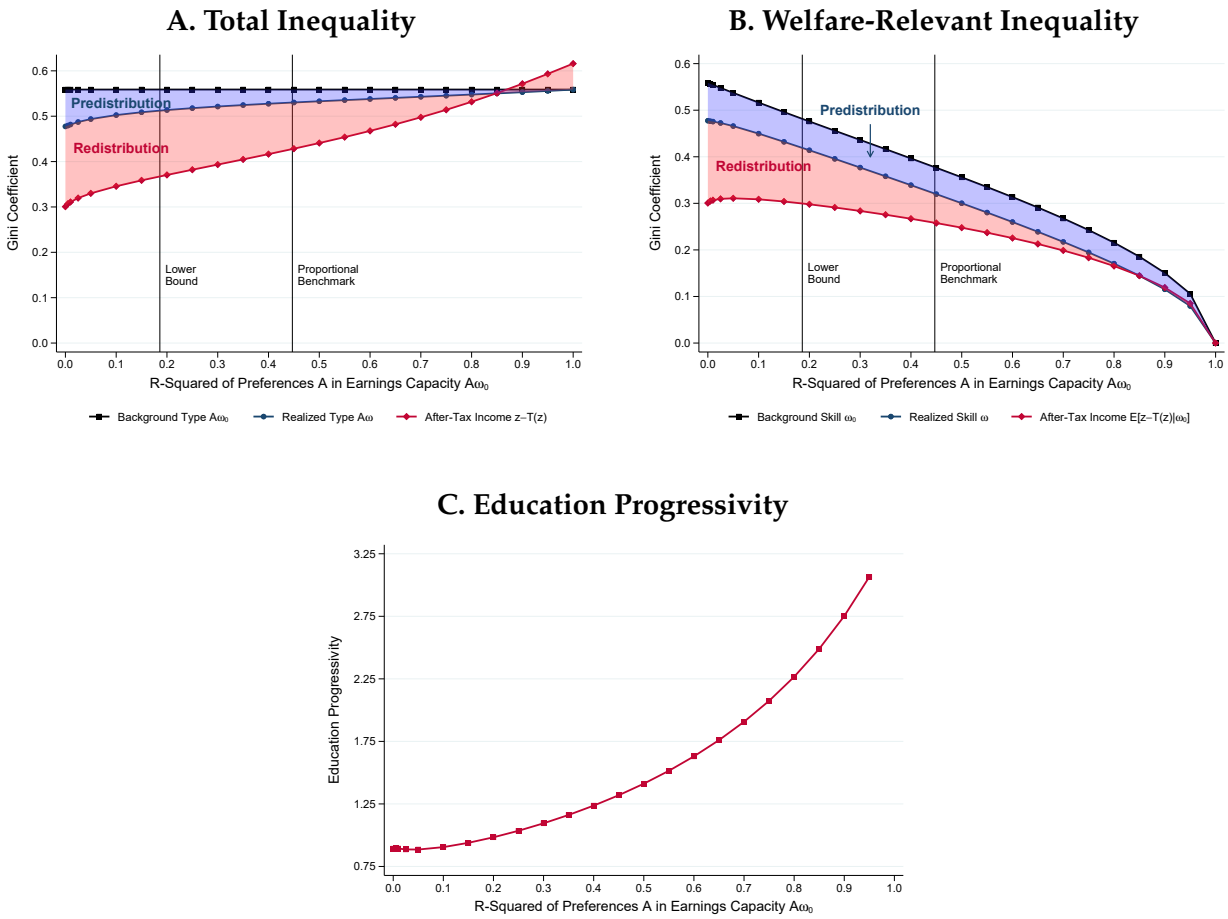
FIGURE 8: INTERGENERATIONAL ELASTICITY OF EARNINGS (IGE) AT THE OPTIMAL POLICY
COMPARATIVE STATICS



Notes: This figure plots the intergenerational elasticity of earnings (IGE) at the optimal policy for different values of the education parameters. Panel A varies the average education elasticity, $\mathbb{E}[\zeta]$, as in Figure 5. Panel B varies heterogeneity in education productivity, measured by the productivity-spread parameter ξ, as in Figure 6.

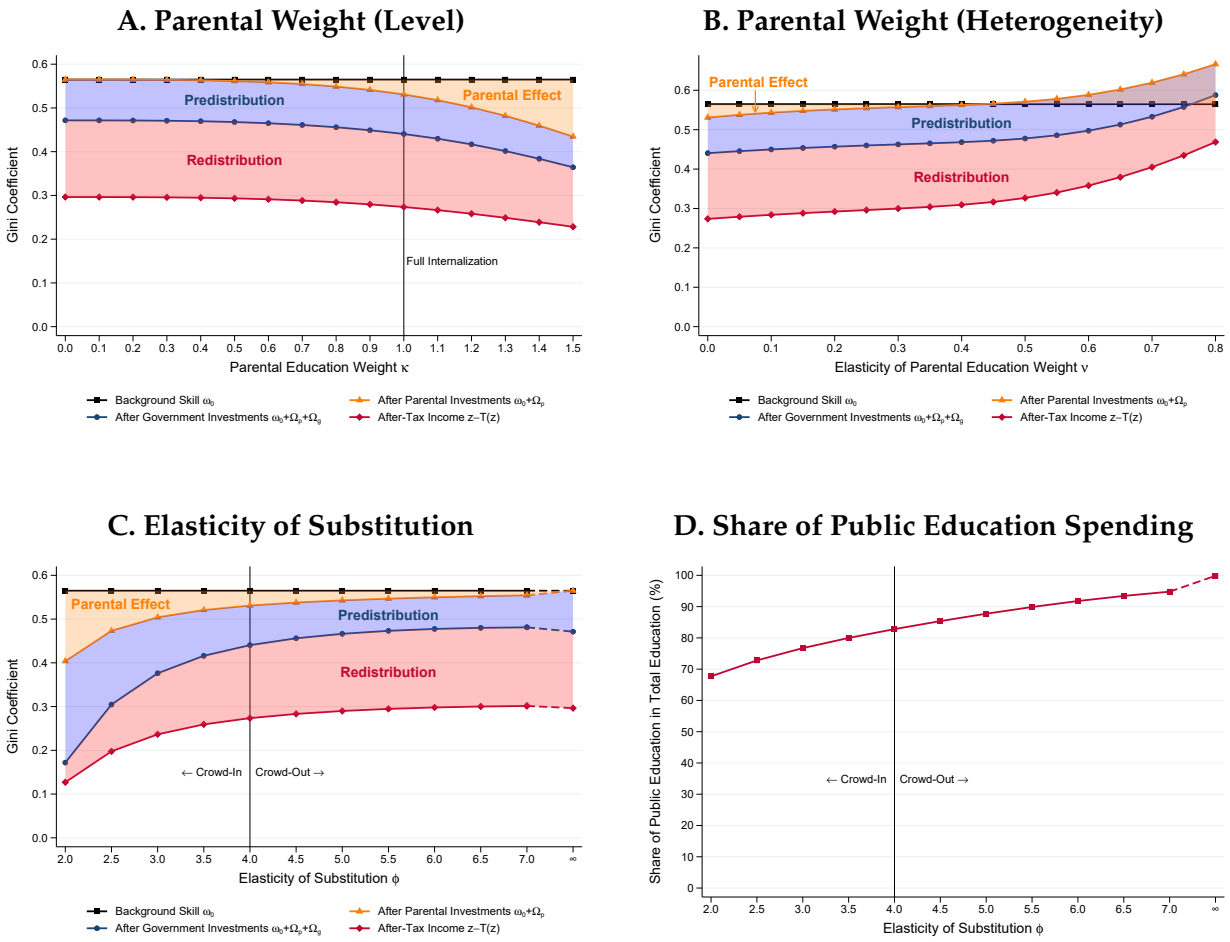
FIGURE 9: EFFECT OF PREFERENCE HETEROGENEITY

DECOMPOSITION OF THE GINI COEFFICIENT



Notes: This figure presents results from numerical simulations with preference heterogeneity. Panel A plots the Gini coefficient of background type $A\omega_0$ (black squares), realized type $A\omega$ (blue circles), and after-tax income $z - T(z)$ (red diamonds) for different values of the relative importance of preference heterogeneity as measured by the R^2 of a regression of $A\omega_0$ against A. The difference between $A\omega_0$ and $A\omega$ reflects predistribution, while the difference between $A\omega$ and $z - T(z)$ reflects redistribution. Panel B is constructed in the same way as Panel A, but focuses on the welfare-relevant part of inequality driven by family-background skill ω_0. Panel C plots the progressivity of education spending as measured by the elasticity of education with respect to background skill in absolute value, i.e. $-d\log E(\omega_0)/d\log\omega_0 = -\mathbb{C}(\log E(\omega_0), \log\omega_0)/\mathbb{V}(\log\omega_0)$.

FIGURE 10: EFFECT OF PARENTAL EDUCATION INVESTMENTS
DECOMPOSITION OF THE GINI COEFFICIENT



Notes: This figure presents results from numerical simulations with parental education investments, using the CES skill production function in equation (47) and abstracting from preference heterogeneity. Panels A-C plot the Gini coefficient of background skill ω_0 (black squares), skill after parental investments $\omega_0 + \Omega_p$ (orange triangles), skill after both parental and government investments $\omega_0 + \Omega_p + \Omega_g$ (blue circles), and finally after-tax income $z - T(z)$ (red diamonds). The difference between ω_0 and $\omega_0 + \Omega_p$ equals the parental effect, the difference between $\omega_0 + \Omega_p$ and $\omega_0 + \Omega_p + \Omega_g$ reflects predistribution, and the difference between $\omega_0 + \Omega_p + \Omega_g$ and $z - T(z)$ reflects redistribution. Panel A varies the parental education weight κ, assuming that the weight is homogeneous across parents. The vertical line at $\kappa = 1$ corresponds to the case where parents fully internalize their children's welfare, while $\kappa = 0$ corresponds to the baseline case without parental investments. Panel B allows the parental weight to increase with family-background skill, the degree to which we parameterize by the elasticity $\nu \equiv d\ln\kappa / d\ln\omega_0$. Panel C fixes $\kappa = 1$ and varies the elasticity of substitution between parental and public investments, $\phi = 1/(1-\delta)$. The vertical line at $\phi = 4$ corresponds to $\delta = \eta = 0.75$, in which case there is neither crowd-in nor crowd-out of parental investments ($\Omega''_{pg} = 0$). Values of ϕ below 4 imply crowd-in, while values above 4 imply crowd-out. The point labeled ∞ on the x-axis corresponds to the limit of perfect substitutability between parental and public investments ($\delta \to 1$). Panel D plots the share of public education spending in total spending for the same values of ϕ as in Panel C. In all panels, the parameters $(B, \mu_{\omega_0}, \sigma_{\omega_0})$ are recalibrated at $\kappa = 1$ and $\delta = \eta$ to match the empirical moments of the baseline calibration (accounting for parental investments) and then held constant across the comparative statics.

Online Appendix

Equality of Opportunity vs Equality of Outcomes: A Theory of Optimal Predistribution

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Danny Yagan

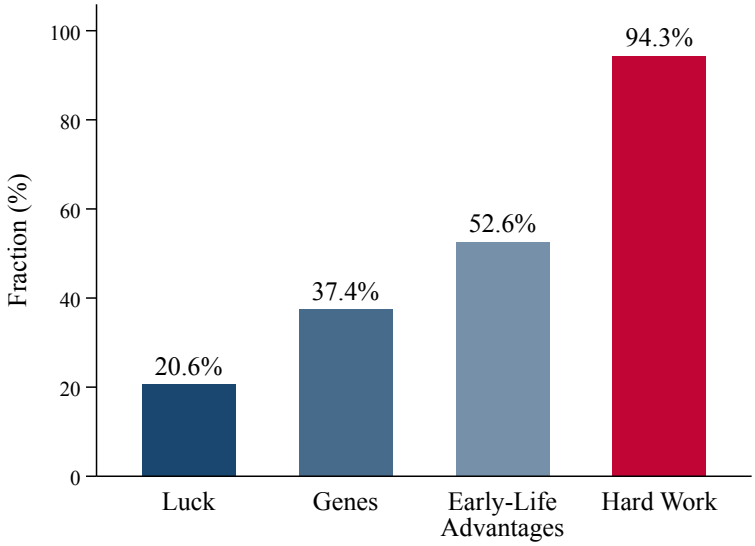
November 2026

A Supplementary Exhibits

FIGURE A.1: BELIEFS ABOUT SUCCESS

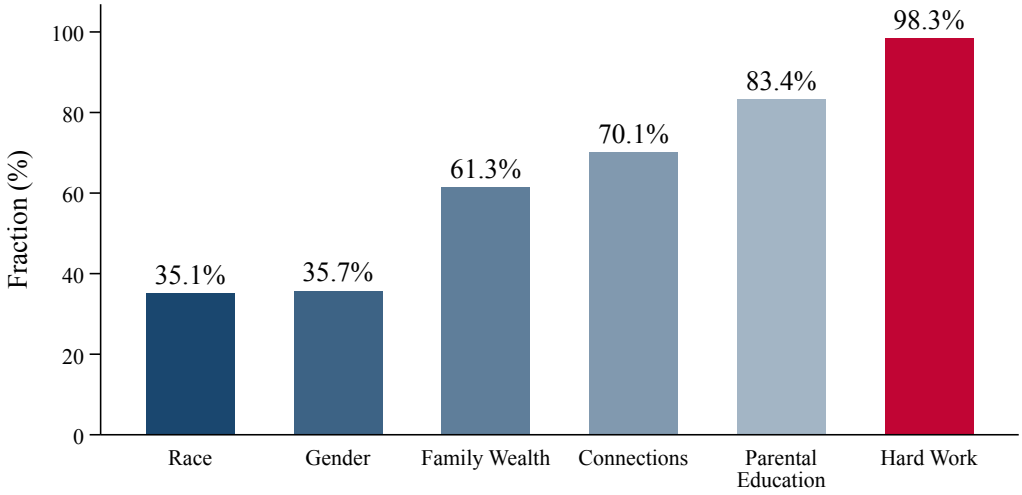
A. 1993 GSS

Why does a person's life turn out well or poorly?



B. 2021 GSS

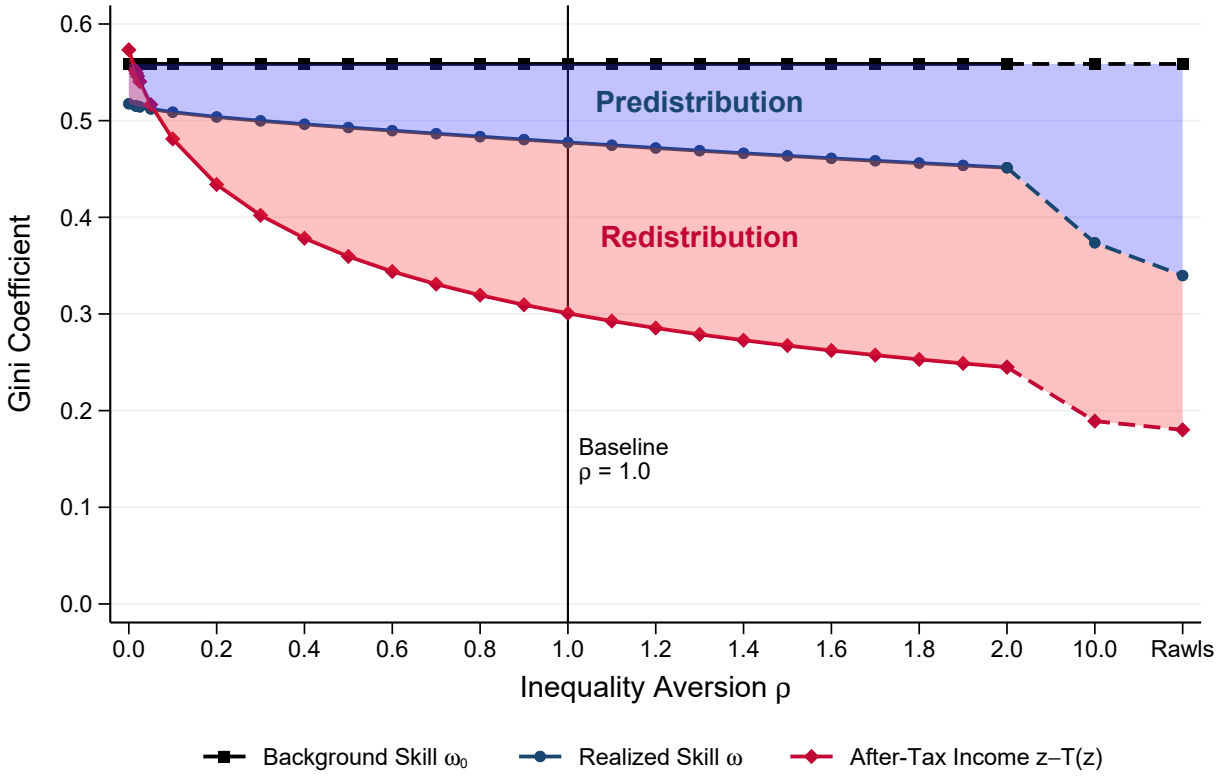
What is important for getting ahead in life?



Notes: This figure reports beliefs about the determinants of success from the 1993 and 2021 waves of the General Social Survey (GSS). Panel A considers a question from the 1993 wave, also used in Figure 1B to measure policy preferences. It shows the share of respondents who rate different factors as either important or very important for "why a person's life turns out well or poorly." A fifth surveyed factor ("such things are decided by God") is omitted. Panel B considers a question from the 2021 wave. It shows the share of respondents who rate different factors as at least fairly important for "getting ahead in life." Race, gender, family wealth, parental education, and hard work correspond to the survey items "a person's race," "being born a man or a woman," "coming from a wealthy family," "having well-educated parents," and "hard work," respectively. The variable labeled "connections" combines two survey items, "knowing the right people" and "having political connections," and is coded as at least fairly important if either item is. Shares are computed using GSS person post-stratification weights.

FIGURE A.2: EFFECT OF INEQUALITY AVERSION

DECOMPOSITION OF THE GINI COEFFICIENT



Notes: This figure plots the Gini coefficient of background skill ω_0 (black squares), realized skill ω (blue circles), and after-tax income $z - T(z)$ (red diamonds) for different values of the inequality-aversion parameter ρ. The difference between ω_0 and ω reflects predistribution, while the difference between ω and $z - T(z)$ reflects redistribution. The point labeled "Rawls" on the x-axis corresponds to the limit where $\rho \to \infty$.

B Beliefs About Equality of Opportunity

To motivate our normative framework, we present evidence on beliefs about equality of opportunity in the United States. Figure 1B considers a question from the 1993 wave of the General Social Survey (GSS)—the only wave in which this specific question was included.[39] The question elicits redistributive preferences: do you favor promoting equal opportunity or equal outcomes?[40] Respondents overwhelmingly support equalizing opportunities (87.5% in favor) over equalizing outcomes (12.5% in favor). These views suggest that Americans prefer "opportunity policies" to traditional redistribution via taxes and transfers.

Appendix Figure A.1, Panel A reports elicited beliefs about success from a different question in the 1993 wave of GSS: why does a person's life turn out well or poorly? The respondents are asked to rate the importance of luck, genes, early-life advantages, and hard work. The first three may be interpreted as dimensions of opportunity—factors outside the individual's control—while the last (hard work) may be interpreted as a choice. The figure shows the share of respondents who consider each factor either "important" or "very important."[41] Virtually everyone believes that hard work is important for success in life. Sizable shares of respondents believe that opportunity factors are important too, particularly early-life advantages, but most people view hard work as the single most important factor. These patterns may explain the strong preference for equalizing opportunities over equalizing outcomes. If hard work is critical for how life turns out, equalizing outcomes through taxes and transfers largely redistributes on the basis of choice-based heterogeneity rather than opportunity-based heterogeneity.

Appendix Figure A.1, Panel B considers a differently worded question about the reasons for success drawn from a more recent wave of the GSS (the 2021 wave). Here respondents are asked to rate the importance of race, gender, connections, family wealth, well-educated parents, and hard work. As before, almost everyone views hard work as important for success, further underpinning the opposition to redistributing outcomes rather than opportunities. Among the different opportunity factors, having connections and well-educated parents are viewed as most important. The similarity between the beliefs reported in Panels A and B of Figure A.1 suggests that beliefs about equality of opportunity have been relatively stable over time. This is reassuring given that the question regarding policy preferences in

[39] While this information may appear somewhat dated, the GSS includes many related questions about beliefs and redistributive preferences that are repeated across waves. In general, responses to such recurring questions are stable over time.

[40] Promoting equal opportunity was described as "allowing everyone to compete for jobs and wealth on a fair and even basis," and promoting equal outcomes was described as "insuring that everyone has a decent standard of living and that there are only small differences in wealth and income between the top and bottom in society."

[41] The exact wording of the GSS question is the following: "I'm going to read some statements that give reasons why a person's life turns out well or poorly. As I read each one, tell me whether you think it is very important, important, somewhat important, or not at all important for how somebody's life turns out? (i) Such things are decided by God, (ii) Some people are born with better genes than others, (iii) Society gives some people a head start and holds others back, (iv) Some people use their will power and work harder than others, and (v) It's just a matter of chance." In the figure, we drop the first category (God decides) and label the third category (society gives some people a head start) as "early-life advantages."

Figure 1B is only available in the 1993 wave.

To summarize, the evidence presented here highlights two facts about beliefs: that outcomes are driven by both choices (hard work) and opportunities—with hard work being the single most important factor—and that equalizing opportunities is preferred to equalizing outcomes. Our theoretical framework allows for both of these features.

C Proofs of Remarks 1 and 2

C.1 Proof of Remark 1

The *actual/realized* indirect utility of an individual of type (A, ω_0) is given by

$$u\left(A,\omega_0\right)=z\left(A\widehat{\Omega}\left(\omega_0\right)\right)-T\left(z\left(A\widehat{\Omega}\left(\omega_0\right)\right)\right)-\frac{A\widehat{\Omega}\left(\omega_0\right)}{1+1/\varepsilon}\left(\frac{z\left(A\widehat{\Omega}\left(\omega_0\right)\right)}{A\widehat{\Omega}\left(\omega_0\right)}\right)^{1+1/\varepsilon}, \tag{C1}$$

where $z\left(A\widehat{\Omega}\left(\omega_0\right)\right)=A\widehat{\Omega}\left(\omega_0\right)\left(1-\tau\left(z\left(A\widehat{\Omega}\left(\omega_0\right)\right)\right)\right)^{\varepsilon}$.

The *imputed* utility used by the planner to evaluate social welfare is given by

$$u\left(A^*,\omega_0\right)=z\left(A^*\widehat{\Omega}\left(\omega_0\right)\right)-T\left(z\left(A^*\widehat{\Omega}\left(\omega_0\right)\right)\right)-\frac{A^*\widehat{\Omega}\left(\omega_0\right)}{1+1/\varepsilon}\left(\frac{z\left(A^*\widehat{\Omega}\left(\omega_0\right)\right)}{A^*\widehat{\Omega}\left(\omega_0\right)}\right)^{1+1/\varepsilon}, \tag{C2}$$

where $A^*=\mathbb{E}\left[A\right]$. Aggregate social welfare is defined as

$$W=\int_A\int_{\omega_0}\Psi\left[u\left(A^*,\omega_0\right)\right]dF_0\left(\omega_0|A\right)dF_A\left(A\right). \tag{C3}$$

In what follows, it will be convenient to rewrite W in terms of individuals indexed by i such that $u_i=u\left(A_i,\omega_{0i}\right)$, $u_i^*=u\left(A^*,\omega_{0i}\right)$, and $W=\int_i\Psi\left[u_i^*\right]di$.

To prove that W is Paretian with respect to individualized lump-sum transfers, consider a set of individual-specific utility perturbations $\Delta=\{\delta_i\}_{\forall i}$ such that the actual utility of individual i equals $u\left(A_i,\omega_{0i}\right)+\delta_i$, and the planner's imputed utility of individual i equals $u\left(A^*,\omega_{0i}\right)+\delta_i$. Given quasi-linearity, the perturbations δ_i can be interpreted as individual-specific lump-sum transfers ($\delta_i>0$) or lump-sum taxes ($\delta_i<0$).

We may then show that, for any two sets $\Delta^j=\left\{\delta_i^j\right\}_{\forall i}$ such that $u\left(A_i,\omega_{0i}\right)+\delta_i^1\geq u\left(A_i,\omega_{0i}\right)+\delta_i^2$ with strict inequality for at least one individual, we have $W\left(\Delta^1\right)>W\left(\Delta^2\right)$ where $W\left(\Delta^j\right)=\int_i\Psi\left[u_i^*+\delta_i^j\right]di$.

The proof is straightforward:

$$\begin{aligned} u\left(A_i,\omega_{0i}\right)+\delta_i^1 \geq u\left(A_i,\omega_{0i}\right)+\delta_i^2 \quad &\Rightarrow \quad \delta_i^1 \geq \delta_i^2 \\ &\Rightarrow \quad u\left(A^*,\omega_{0i}\right)+\delta_i^1 \geq u\left(A^*,\omega_{0i}\right)+\delta_i^2 \\ &\Rightarrow \quad \Psi\left[u\left(A^*,\omega_{0i}\right)+\delta_i^1\right] \geq \Psi\left[u\left(A^*,\omega_{0i}\right)+\delta_i^2\right], \end{aligned}$$

given $\Psi[.]$ is a strictly increasing function. If this holds for every individual i, then $W\left(\Delta^1\right) = \int_i \Psi\left[u\left(A^*,\omega_{0i}\right)+\delta_i^1\right] di \geq \int_i \Psi\left[u\left(A^*,\omega_{0i}\right)+\delta_i^2\right] di = W\left(\Delta^2\right)$. Moreover, if $u\left(A_i,\omega_{0i}\right)+\delta_i^1 > u\left(A_i,\omega_{0i}\right)+\delta_i^2$ for at least one individual, then $W\left(\Delta^1\right) > W\left(\Delta^2\right)$ holds with strict inequality.

These derivations prove Remark 1: Given a distribution of types (A,ω_0), the social welfare function is Paretian with respect to any combination of individual-specific utility perturbations (thus allowing for lump-sum taxes and transfers based on type). We may also prove the converse: any non-Paretian welfare effects must involve manipulating the structure of primitives.

To see this, suppose W is non-Paretian. Then there exist allocations $\left\{A_i^1,\omega_{0i}^1,\delta_i^1\right\}_{\forall i}$ and $\left\{A_i^2,\omega_{0i}^2,\delta_i^2\right\}_{\forall i}$ such that $u\left(A_i^1,\omega_{0i}^1\right)+\delta_i^1 \geq u\left(A_i^2,\omega_{0i}^2\right)+\delta_i^2$ for every individual i (Allocation 1 Pareto dominates), yet $\Psi\left[u\left(A^*,\omega_{0i}^1\right)+\delta_i^1\right] < \Psi\left[u\left(A^*,\omega_{0i}^2\right)+\delta_i^2\right]$ for at least one individual (such that Allocation 1 may generate a lower value of W). Hence,

$$\begin{aligned} \Psi\left[u\left(A^*,\omega_{0i}^1\right)+\delta_i^1\right] < \Psi\left[u\left(A^*,\omega_{0i}^2\right)+\delta_i^2\right] \quad &\Rightarrow \quad u\left(A^*,\omega_{0i}^1\right)+\delta_i^1 < u\left(A^*,\omega_{0i}^2\right)+\delta_i^2 \\ &\Rightarrow \quad u\left(A^*,\omega_{0i}^1\right)+\delta_i^1+u\left(A_i^1,\omega_{0i}^1\right)-u\left(A_i^1,\omega_{0i}^1\right) < u\left(A^*,\omega_{0i}^2\right)+\delta_i^2 \\ &\Rightarrow \quad u\left(A^*,\omega_{0i}^1\right)+u\left(A_i^2,\omega_{0i}^2\right)+\delta_i^2-u\left(A_i^1,\omega_{0i}^1\right) < u\left(A^*,\omega_{0i}^2\right)+\delta_i^2 \\ &\Rightarrow \quad u\left(A_i^2,\omega_{0i}^2\right)-u\left(A_i^1,\omega_{0i}^1\right) < u\left(A^*,\omega_{0i}^2\right)-u\left(A^*,\omega_{0i}^1\right). \end{aligned}$$

Now, assume that the allocations are constrained to be fixed in primitives, i.e., $A_i^1 = A_i^2$ and $\omega_{0i}^1 = \omega_{0i}^2$ for any individual i. From the inequality derived above, this constraint generates a contradiction: $0 < 0$. Therefore, any change in allocation that involves non-Paretian behavior of the social welfare function must be associated with a reallocation of the primitives of the model.

C.2 Proof of Remark 2

The proof of the first part of Remark 2 is as follows. Assume that $A^* \in \text{supp}(A|\omega_0)$ for all ω_0. Consider two policy environments (T^1,E^1) and (T^2,E^2). Let actual/realized indirect utilities in the different policy regimes be given by $u^1(A,\omega_0)$ and $u^2(A,\omega_0)$. Let imputed indirect utilities be given by $u^1(A^*,\omega_0)$ and $u^2(A^*,\omega_0)$. We denote social welfare in each environment as W^1 and W^2. We want to show that, if

$u^2(A,\omega_0) \geq u^1(A,\omega_0)$ for all (A,ω_0) in the population support, then $W^2 \geq W^1$. If $A^* \in \text{supp}(A|\omega_0)$ for all ω_0, then $u^2(A,\omega_0) \geq u^1(A,\omega_0)$ for all (A,ω_0) implies that it is true in particular for the combinations (A^*,ω_0), so that $u^2(A^*,\omega_0) \geq u^1(A^*,\omega_0)$, for all ω_0. Because Ψ is strictly increasing, it follows that $W^2 \geq W^1$.

This result departs from strict Paretianism because there might be cases where $u^2(A,\omega_0) \geq u^1(A,\omega_0)$ for all (A,ω_0), $u^2(A,\omega_0) > u^1(A,\omega_0)$ for some (A,ω_0), and $W^2 = W^1$. A simple example is as follows. Consider an individual with large $A = \widetilde{A}$ so that there is no $\widetilde{\omega}_0$ in the relevant support of ω_0 such that $\widetilde{A}\widetilde{\omega}_0 = A^*\omega_0$, for some realized ω_0. These cases always exist if the supports of A and ω_0 are bounded. In such a case, a small tax cut on this individual will generate a strict increase in her actual utility with potentially no effect on social welfare. We note, however, that these cases cannot arise solely from differences in education policies, as $E(\omega_0)$ is targeted at the welfare-relevant primitive and, therefore, always affects actual and imputed utilities in the same direction.

Using this intuition, we proceed to prove the second part of Remark 2 in two steps. First, we present a lemma that characterizes the policies that induce these violations of strict Paretianism. Then, we show that they never coincide with the social optimum.

Lemma 1. *Assume that the support of ω_0 is connected; $A^* \in \text{int}\,\text{supp}(A|\omega_0)$ for all ω_0; the individual problem admits a unique solution in z; and $\widehat{\Omega}(\omega_0)$ is continuous and strictly positive for all ω_0. Then, any two policy environments (T^1,E^1) and (T^2,E^2) whose comparison violates the strict Pareto principle satisfy $\Omega(\omega_0,E^2(\omega_0)) = k\Omega(\omega_0,E^1(\omega_0))$ for all ω_0, where $k>0$ is a constant that does not depend on ω_0.*

Proof. Consider an arbitrary type (A,ω_0) for which $u^2(A,\omega_0) > u^1(A,\omega_0)$ but $u^2(A^*,\omega_0) = u^1(A^*,\omega_0)$. It is straightforward to see that this type has $A \neq A^*$. We write indirect utility as $u^j(A,\omega_0) = \max_z \left\{ z - T^j(z) - \frac{A\omega_j(\omega_0)}{1+1/\varepsilon}\left(\frac{z}{A\omega_j(\omega_0)}\right)^{1+1/\varepsilon} \right\}$, where $\omega_j(\omega_0) = \Omega(\omega_0, E^j(\omega_0))$. We can conveniently write $u^j = C_j - D_j$, where C_j is the consumption part of indirect utility $C_j = z_j - T^j(z_j)$ and D_j is the effort-disutility part of indirect utility $D_j = \frac{A\omega_j(\omega_0)}{1+1/\varepsilon}\left(\frac{z_j}{A\omega_j(\omega_0)}\right)^{1+1/\varepsilon}$. We denote by $u^{j*} = C_j^* - D_j^*$ the imputed indirect utility evaluated at $A = A^*$.

Fix ω_0. Define $g(A) = u^2(A,\omega_0) - u^1(A,\omega_0)$. We know that $g(A) \geq 0$ for all A, that there is some A for which $g(A) > 0$, and that $g(A^*) = 0$. Because $A^* \in \text{int}\,\text{supp}(A|\omega_0)$, we know that $g'(A^*) = 0$. Using the envelope theorem, we know that $\partial u^j/\partial A = -\partial D_j/\partial A = D_j/\varepsilon A$. Then, $g'(A^*) = 0$ implies $D_1^* = D_2^*$, which in turn implies that $C_1^* = C_2^*$. That is, whenever there is a tie in social welfare despite a strict individual gain, it is because the reference type consumes the same and has the same disutility of effort in both environments. Said differently, a case where the two policies leave the reference type indifferent because extra consumption compensates for extra effort cannot lead to ties that depart from strong Pareto.

We also note that, for $W^2 = W^1$ to hold, we need $u^{1*} = u^{2^*}$ for all ω_0, given that $\Psi' > 0$. It follows

that $\partial g(A^*)/\partial\omega_0 = 0$. Again, using the envelope theorem, we know that $\partial u^{j*}/\partial\omega_0 = -\partial D_j^*/\partial\omega_0 = (D_j^*/\varepsilon)\cdot(\omega_j'(\omega_0)/\omega_j(\omega_0))$. Above we showed that $D_1^* = D_2^*$, so the previous derivative implies $\omega_2'(\omega_0)/\omega_2(\omega_0) = \omega_1'(\omega_0)/\omega_1(\omega_0)$. Because $\omega_j(\omega_0) > 0$ for all j, ω_0 and the support of ω_0 is connected, we can conclude that $\omega_2(\omega_0) = k\omega_1(\omega_0)$, with $k > 0$ a constant that does not depend on ω_0. This follows from noting that $\omega_2'(\omega_0)/\omega_2(\omega_0) = \omega_1'(\omega_0)/\omega_1(\omega_0)$ implies $d/d\omega_0 \log(\omega_2(\omega_0)/\omega_1(\omega_0)) = 0$, which implies that $\log(\omega_2(\omega_0)/\omega_1(\omega_0)) = K$, with K a constant. Defining $k = \exp(K)$ yields the result. Then, in cases that induce ties, the differences between E^1 and E^2 (if any) make a proportional change in skills along the distribution of ω_0: the two education policies must satisfy $\Omega(\omega_0, E^2(\omega_0)) = k\Omega(\omega_0, E^1(\omega_0))$ for all ω_0. □

Using this result, we proceed to show that these cases never coincide with the optimum under the assumption that $\Omega(\omega_0, E(\omega_0))$ is strictly concave in $E(\omega_0)$. Towards a contradiction, consider two policies 1 and 2. Assume policy 1 is optimal and that policy 2 is feasible and strictly Pareto-dominates policy 1. Because of the first part of Remark 2, we know that if this happens, then $W^1 = W^2$ (as weak Pareto implies that strictly Pareto dominated policies cannot be strictly preferred). The lemma above shows that these cases imply that $\omega_2(\omega_0) = k\omega_1(\omega_0)$. We write $n = A\omega_1(\omega_0)$ and define $k_1 = 1$ and $k_2 = k$. This allows us to write indirect utilities in both environments as a function of the initial capacity n:

$$u^1(A,\omega_0) = z_1(n) - T^1(z_1(n)) - \frac{\varepsilon}{1+\varepsilon}\frac{z_1(n)^{1+1/\varepsilon}}{n^{1/\varepsilon}}, \tag{C4}$$

$$u^2(A,\omega_0) = z_2(kn) - T^2(z_2(kn)) - \frac{\varepsilon}{1+\varepsilon}\frac{z_2(kn)^{1+1/\varepsilon}}{(kn)^{1/\varepsilon}}. \tag{C5}$$

Then, we define $c_j(n) = z_j(k_j n) - T^j(z_j(k_j n))$ as the consumption index of type n in regime j and $x_j(n) = \frac{z_j(k_j n)^{1+1/\varepsilon}}{k_j^{1/\varepsilon}}$ as the effort index. Using $(k_j n)^{1/\varepsilon} = k_j^{1/\varepsilon} n^{1/\varepsilon}$, we can write $V_j(n) = c_j(n) - \frac{\varepsilon}{1+\varepsilon}\frac{x_j(n)}{n^{1/\varepsilon}}$, so that indirect utilities are indexed at the initial earnings capacity. Indirect utility is linear in both the consumption index $c_j(n)$ and the effort index $x_j(n)$, with the same type coefficient in both environments (so this is equivalent to comparing the (A,ω_0) type across the policy environments).

Now we build an intermediate incentive-compatible allocation. For $0 < \theta < 1$, define $x_\theta = (1-\theta)x_1 + \theta x_2$ and $c_\theta = (1-\theta)c_1 + \theta c_2$. These intermediate allocations are incentive compatible. Formally, define $a(n) = \frac{\varepsilon}{1+\varepsilon}\frac{1}{n^{1/\varepsilon}}$, so that we can write $v_j(n) = c_j(n) - a(n)x_j(n)$. Because policy 1 is optimal and policy 2 is feasible, both are incentive compatible. This means that:

$$c_1(n) - a(n)x_1(n) \geq c_1(m) - a(n)x_1(m), \tag{C6}$$

$$c_2(n) - a(n)x_2(n) \geq c_2(m) - a(n)x_2(m), \tag{C7}$$

for all n, m. Multiplying the first inequality by $1-\theta$, the second by θ, and adding, gives $c_\theta(n) - a(n)x_\theta(n) \geq c_\theta(m) - a(n)x_\theta(m)$, for all n, m. This intermediate allocation can be implemented with the tax function $T^\theta(z_\theta(n)) = z_\theta(n) - c_\theta(n)$.

Define $u_\theta(n) = (1-\theta)u_1(n) + \theta u_2(n)$. Because $W^1 = W^2$, it follows that for reference individuals $u_1(A^*, \omega_0) = u_2(A^*, \omega_0) = u_\theta(A^*, \omega_0)$. Then, $W^\theta = W^1 = W^2$.

Now we show that the allocation θ generates more resources. First, consider intermediate skills $\omega_\theta(\omega_0) = k_\theta \omega_1(\omega_0)$, with $k_\theta = (1-\theta) + \theta k$. This implies that the earnings needed to implement the effort index x_θ are given by $z_\theta = k_\theta^{1/(1+\varepsilon)} x_\theta^{\varepsilon/(1+\varepsilon)}$. Note that z_θ is a concave function of (k_θ, x_θ), which implies that $z_\theta \geq (1-\theta)z_1 + \theta z_2$.

Consider two cases. First, assume $E^1 \neq E^2$. Define E^θ as the solution to $\Omega(\omega_0, E^\theta(\omega_0)) = (1-\theta)\omega_1(\omega_0) + \theta\omega_2(\omega_0) = (1-\theta+\theta k)\omega_1(\omega_0)$, which exists because Ω is continuous and strictly increasing. Then, the strict concavity of Ω implies that $E^\theta(\omega_0) < (1-\theta)E^1(\omega_0) + \theta E^2(\omega_0)$. This result implies that:

$$\begin{aligned}\int (z_\theta - c_\theta)dF - p\int E^\theta dF \quad &> \quad (1-\theta)\left\{\int (z_1 - c_1)dF - p\int E^1 dF\right\} \\ &\quad +\theta\left\{\int (z_2 - c_2)dF - p\int E^2 dF\right\}. \end{aligned} \tag{C8}$$

Since policy 1 is optimal and policy 2 is feasible, both exhaust the budget (if not, a common reduction in the tax intercept would leave earnings unchanged and strictly increase the utility of reference types, contradicting that policy 1 is optimal). We conclude that:

$$\int (z_\theta - c_\theta)dF - p\int E_\theta dF > R. \tag{C9}$$

Then, the intermediate policy θ yields the same welfare as the optimal policy but generates a surplus in terms of revenue.

Conversely, assume $E^1 = E^2$, so that $E^\theta(\omega_0) = (1-\theta)E^1(\omega_0) + \theta E^2(\omega_0)$. In this case $k = 1$. With $k = 1$, we have that $z_j = x_j^{\varepsilon/(1+\varepsilon)}$, which is a strictly concave function. So, if for at least one type $x_1 \neq x_2$, then $z_\theta > (1-\theta)z_1 + \theta z_2$ for that type, and the budget constraint integral is strictly larger than R. Now we show that $x_1 = x_2$ cannot be true for all types. If $x_1 = x_2$ for all types, then $z_1 = z_2$ for all types. But then the strict Pareto improvement would require $c_2 > c_1$ for some types, which is a contradiction with budget balance if $E^1 = E^2$. Then, for at least one type, $x_1 \neq x_2$ and, therefore, the strict inequality holds.

The planner can then rebate that surplus to everyone. Because utility is quasi-linear, that surplus does not affect earnings and increases everyone's utility, including the reference types, thus increasing social welfare. Then this feasible policy generates strictly more social welfare than policies 1 and 2, a contradiction with policy 1 being optimal. We conclude that the policy comparisons that violate the strict

Pareto principle are not relevant under the optimal policy.

D General Dynamic Framework: Optimal Policy

Consider the dynamic framework laid out in Section 3.3. To solve for the optimal policies in this model, we start by defining social marginal welfare weights. Given tax and education schedules depend on income, it is convenient to express welfare weights in terms of income levels. Specifically, we define

$$g_t\left(z_t, A_t, A_t^*\right) \equiv \frac{\delta_t \Psi' \left[u_t\left(\frac{A_t^*}{A_t} z_t, A_t^* \omega_t\left(z_t, A_t\right)\right)\right]}{\beta^t \lambda}, \tag{D1}$$

as the social marginal welfare weight at earnings z_t and preferences A_t in generation t, evaluated at average preferences A_t^*. Correspondingly, we denote the average welfare weight at earnings level z_t by $g_t\left(z_t, A_t^*\right)$ and the average welfare weight *above* earnings level z_t by $G_t\left(z_t, A_t^*\right)$.

To characterize the optimal tax schedule in generation t, we consider a small reform which changes the marginal tax rate by $d\tau_t$ on the earnings interval $(z_t, z_t + dz_t)$. As in the static model, such a reform has three welfare effects: a mechanical revenue effect dM_{τ_t}, a behavioral revenue effect dB_{τ_t}, and a direct welfare effect dW_{τ_t}. The three effects may be derived as follows.

The mechanical revenue effect (evaluated at the discounted marginal value of public funds $\beta^t \lambda$) is given by

$$dM_{\tau_t} = \beta^t \lambda \cdot d\tau_t \cdot dz_t \cdot \left(1 - H_t\left(z_t\right)\right), \tag{D2}$$

where $H_t\left(z_t\right)$ denotes the unconditional distribution function for earnings in generation t.

The behavioral revenue effect (also evaluated at the discounted marginal value of public funds $\beta^t \lambda$) is given by

$$\begin{aligned} dB_{\tau_t} &= \beta^t \lambda \cdot \left[\tau_t\left(z_t\right) - pe_t\left(z_t\right)\right] \cdot dz_t \cdot h_t\left(z_t\right) dz_t \\ &= -\beta^t \lambda \cdot \left[\frac{\tau_t\left(z_t\right) - pe_t\left(z_t\right)}{1 - \tau_t\left(z_t\right)}\right] \cdot \varepsilon \cdot z_t \cdot d\tau_t \cdot h_t\left(z_t\right) dz_t, \end{aligned} \tag{D3}$$

where $e_t\left(z_t\right) \equiv E_t'\left(z_t\right)$ denotes the marginal effect of earnings on education provision, and $h_t\left(z_t\right)$ denotes the unconditional density function for earnings in generation t.

Finally, the direct welfare effect can be written as

$$dW_{\tau_t} = -\int_{A_t} \int_{z_t}^{\overline{z}_t} \delta_t \Psi' \left[.\right] \cdot d\tau_t \cdot dz_t \cdot dF_{z_t}\left(z_t' | A_t\right) dF_{A_t}\left(A_t\right) = -G_t\left(z_t, A_t^*\right) \cdot dM_{\tau_t}. \tag{D4}$$

We may then characterize optimal redistribution as follows:

Proposition 6 (Optimal Redistribution: Dynamic Model). *In each generation t and at each income level z_t, the optimal marginal tax rate satisfies*

$$\frac{\tau_t(z_t) - pe_t(z_t)}{1 - \tau_t(z_t)} = \frac{1 - G_t(z_t, A_t^*)}{\varepsilon \cdot \alpha_t(z_t)}, \tag{D5}$$

where $e_t(z_t) \equiv E_t'(z_t)$ denotes marginal education spending in generation t, ε is the elasticity of earnings with respect to the marginal net-of-tax rate, $\alpha_t(z_t) \equiv z_t h_t(z_t) / (1 - H_t(z_t))$ is the local Pareto parameter of the earnings distribution in generation t, and $G_t(z_t, A_t^*)$ is the average social marginal welfare weight on workers above income level z_t in generation t.

Proof. The result follows from the condition that $dM_{\tau_t} + dB_{\tau_t} + dW_{\tau_t} = 0$ in each generation t. Inserting equations (D2)-(D4) and rearranging terms gives the optimal tax rule in equation (D5). □

The dynamic optimal tax rule in Proposition 6 is similar to the static rule in Proposition 1. The key difference is the term $pe_t(z_t)$ on the left-hand side of equation (D5), creating a direct link between education provision and tax policy. This term captures a fiscal externality of earnings responses operating through education spending on the next generation. If education provision is progressive ($e_t(z_t) < 0$), then lower earnings in generation t imply higher education spending on generation $t+1$, thereby increasing the fiscal externality of raising taxes. All else equal, the extra term in the optimal tax rule implies that, within a generation, redistribution and predistribution are *policy substitutes*: greater progressivity in education spending implies lower progressivity in income taxation.

To characterize the optimal education schedule in generation t, we consider a small reform which changes education provision by dE_t on the earnings interval $(z_t, z_t + dz_t)$. This affects skill levels in generations $s = t+1, ..., T$ among those descending from individuals in the targeted income range in generation t. The mass of affected individuals in each generation s equals the mass of targeted individuals in generation t, i.e. $h_t(z_t)\, dz_t$. Again, we have three welfare effects of the reform: a mechanical revenue effect dM_{E_t}, a behavioral revenue effect dB_{E_t}, and a direct welfare effect dW_{E_t}.

The mechanical revenue effect equals

$$dM_{E_t} = -\beta^t \lambda \cdot p \cdot dE_t \cdot h_t(z_t)\, dz_t. \tag{D6}$$

The behavioral revenue effect can be written as

$$dB_{E_t} = \sum_{s=t+1}^{T} \beta^s \lambda \cdot \mathbb{E}\left[(\tau_s(z_s) - pe_s(z_s)) \cdot \frac{dz_s}{d\omega_s} \cdot \widehat{\Omega}'_{E_t} \mid z_t \right] \cdot dE_t \cdot h_t(z_t)\, dz_t, \tag{D7}$$

where the expectation $\mathbb{E}[.]$ in each generation s is conditional on descending from individuals at the

targeted income level z_t in generation t.

Finally, the direct welfare effect can be written as

$$dW_{E_t} = \sum_{s=t+1}^{T} \delta_s \cdot \mathbb{E}\left[\Psi'\left[.\right] \cdot \frac{du_s}{d\omega_s} \cdot \widehat{\Omega}'_{E_t} \mid z_t\right] \cdot dE_t \cdot h_t\left(z_t\right) dz_t. \tag{D8}$$

These derivations allow us to characterize optimal predistribution:

Proposition 7 (Optimal Predistribution: Dynamic Model). *In each generation t and at each income level z_t, optimal education provision satisfies*

$$\sum_{s=t+1}^{T} \beta^{s-t} \mathbb{E}\left[\widehat{\Omega}'_{E_t} \cdot \left(g_s\left(z_s, A_s, A_s^*\right) \cdot \frac{du_s}{d\omega_s} + \left(\tau_s\left(z_s\right) - pe_s\left(z_s\right)\right) \cdot \frac{dz_s}{d\omega_s}\right) \mid z_t\right] = p, \tag{D9}$$

where $e_s\left(z_s\right) \equiv E_s'\left(z_s\right)$ denotes marginal education spending in generation s, $du_s/d\omega_s$ is the marginal willingness to pay for higher skill in generation s, and $g_s\left(z_s, A_s, A_s^*\right)$ is the social marginal welfare weight at earnings z_s and preferences A_s in generation s.

Proof. The result follows from the condition that $dM_{E_t} + dB_{E_t} + dW_{E_t} = 0$ in each generation t. Inserting equations (D6)-(D8) and rearranging terms gives the optimal education rule in equation (D9). □

Proposition 7 generalizes Proposition 2 to allow for intergenerational dynamics. The dynamic rule differs from the static rule in two respects. First, the social marginal benefit of education provision in generation t (left-hand side of equation D9) is a present discounted value of effects in generations $t+1, ..., T$. The trajectory of effects over time depends on the strength of intergenerational skill transmission embodied in the function $\widehat{\Omega}\left(.\right)$. Second, the social marginal benefit of education provision accounts for the fact that earnings responses to higher skill in future generations create fiscal externalities through education spending. This effect is captured by the term $pe_s\left(z_s\right)$ on the left-hand side of equation (D9).

E Simulation Details

This appendix describes all simulation details. It first describes the baseline case with no preference heterogeneity, and then moves to the cases with preference heterogeneity and parental investments.

E.1 Simulations Without Preference Heterogeneity

E.1.1 Preliminaries and Main Equations

We start by assuming no preference heterogeneity (i.e., assuming a constant A). Note that abstracting from preference heterogeneity affects optimal policy but not the calibration procedure, so the same

calibrated values are valid for the simulations with preference heterogeneity. For completeness, we present derivations for an arbitrary constant A, although the baseline simulations normalize, without loss of generality, A to 1.[42] Then, the utility function is given by

$$u\left(z;\omega_0\right)=z-T\left(z\right)-\frac{A\widehat{\Omega}\left(\omega_0\right)}{1+1/\varepsilon}\left(\frac{z}{A\widehat{\Omega}\left(\omega_0\right)}\right)^{1+1/\varepsilon}. \tag{E1}$$

Optimal earnings obey

$$z\left(\omega_0\right)=A\widehat{\Omega}\left(\omega_0\right)\left(1-\tau\left(z\left(\omega_0\right)\right)\right)^{\varepsilon}, \tag{E2}$$

with $\tau\left(z\right)=T'\left(z\right)$. Indirect utility is given by $U\left(\omega_0\right)=\max_z u\left(z;\omega_0\right)=u\left(z\left(\omega_0\right);\omega_0\right)$.

Realized skill $\widehat{\Omega}\left(\omega_0\right)$ is given by

$$\widehat{\Omega}\left(\omega_0\right)=\Omega\left(\omega_0,E\left(\omega_0\right)\right)=\omega_0+B\cdot E\left(\omega_0\right)^{\eta}, \tag{E3}$$

with ω_0 background skill. Background skill is distributed across individuals with PDF f. We assume ω_0 is distributed log-normal with parameters $\left(\mu_{\omega_0},\sigma_{\omega_0}\right)$, and assume that ω_0 has finite support $[\underline{\omega}_0,\overline{\omega}_0]$, with $\underline{\omega}_0>0$ and $\overline{\omega}_0<\infty$.[43] Under this utility function, ω_0 is interpreted as potential earnings in the absence of policy (taxes and education).

The government provides $E\left(\omega_0\right)$ at a revenue cost of p per unit, so the government budget constraint is given by

$$\int_{\omega_0}T\left(z\left(\omega_0\right)\right)f\left(\omega_0\right)d\omega_0=R+p\int_{\omega_0}E\left(\omega_0\right)f\left(\omega_0\right)d\omega_0, \tag{E4}$$

where R is an exogenous revenue requirement. Implicit in this formulation is the normalization of the population size to 1. We denote the Lagrange multiplier associated with the government budget constraint by λ.

The social welfare function is given by

$$W=\int_{\omega_0}\Psi\left[U\left(\omega_0\right)\right]f\left(\omega_0\right)d\omega_0. \tag{E5}$$

In the simulations, we assume $\Psi\left[U\right]=U^{1-\rho}/\left(1-\rho\right)$, with ρ a measure of inequality aversion.

The solution to the optimal policy problem is characterized by the optimality conditions for education

[42] This normalization is without loss of generality in the constant-A case because any constant A will yield the same results after a corresponding recalibration of μ_{ω_0} and B, defined below.

[43] While the log-normal distribution is unbounded from above and defined over all positive numbers, we operationalize this by choosing, for given $\left(\mu_{\omega_0},\sigma_{\omega_0}\right)$, $\underline{\omega}_0$ and $\overline{\omega}_0$ so that $F\left(\underline{\omega}_0\right)=0.00001$ and $F\left(\overline{\omega}_0\right)=0.99999$.

$$A\left(1-\tau\left(z\left(\omega_0\right)\right)\right)^{\varepsilon}\left(\tau\left(z\left(\omega_0\right)\right)+g\left(\omega_0\right)\frac{1-\tau\left(z\left(\omega_0\right)\right)}{1+\varepsilon}\right)B\eta\left(E\left(\omega_0\right)\right)^{\eta-1}=p, \tag{E6}$$

for all ω_0, where $g\left(\omega_0\right)=U\left(\omega_0\right)^{-\rho}/\lambda$ is the social marginal welfare weight of individuals of type ω_0; the optimality conditions for marginal tax rates

$$\frac{\tau\left(z\left(\omega_0\right)\right)}{1-\tau\left(z\left(\omega_0\right)\right)}=\frac{1}{\varepsilon}\cdot\frac{\widehat{\Omega}'\left(\omega_0\right)}{\widehat{\Omega}\left(\omega_0\right)f\left(\omega_0\right)}\cdot\int_{\omega_0}^{\infty}\left(1-g\left(s\right)\right)f\left(s\right)ds, \tag{E7}$$

for all ω_0, where $\widehat{\Omega}'\left(\omega_0\right)=1+B\eta\left(E\left(\omega_0\right)\right)^{\eta-1}E'\left(\omega_0\right)$ is the Jacobian of the change of variables; and the budget constraint (equation E4) that pins down the intercept of the tax system $T\left(\underline{\omega}_0\right)$ given marginal tax rates because of the fundamental theorem of calculus (as is standard, we operationalize this as a universal lump-sum transfer plus a sequence of marginal tax rates). To understand why $\widehat{\Omega}'\left(\omega_0\right)$ shows up in equation (E7) and A does not, define $\hat{\omega}=A\omega$. Proposition 1 is written in terms of earnings. To express it in terms of types, the translation involves two changes of variables: from z to $\hat{\omega}$, and from $\hat{\omega}$ to ω_0. We use the local linearization approach of Saez (2001). Assuming $\tau'\left(z\right)=0$, equation (E2) yields $\hat{\omega}\frac{dz}{d\hat{\omega}}=z$. Then, the identity $h\left(z\right)dz=f_{\hat{\omega}}\left(\hat{\omega}\right)d\hat{\omega}$ implies $\hat{\omega}f_{\hat{\omega}}\left(\hat{\omega}\right)=\hat{\omega}h\left(z\right)\frac{dz}{d\hat{\omega}}=zh\left(z\right)$. Then, when A is constant, we have that $\frac{d\hat{\omega}}{d\omega_0}=A\widehat{\Omega}'\left(\omega_0\right)$. Then, from the identity $f\left(\omega_0\right)d\omega_0=f_{\hat{\omega}}\left(\hat{\omega}\right)d\hat{\omega}$, we get that $f\left(\omega_0\right)=f_{\hat{\omega}}\left(\hat{\omega}\right)A\widehat{\Omega}'\left(\omega_0\right)$, so that $zh\left(z\right)=\hat{\omega}\frac{f(\omega_0)}{A\omega'(\omega_0)}=\frac{\widehat{\Omega}(\omega_0)f(\omega_0)}{\widehat{\Omega}'(\omega_0)}$, which gives the result. Note that, when education policy is non-existent, as in the standard Mirrleesian model, we get that $zh\left(z\right)=\omega_0 f\left(\omega_0\right)$, which would be the standard result given the assumed utility function.

E.1.2 Discretization

We work with a discretized type distribution. To tractably over-sample the bottom part of the distribution (where most of the log-normal mass is concentrated), we simulate an equally-spaced grid of a standard normal variable $y^i\in y^1,...,y^N$, with bin width $\Delta_y=y^i-y^{i-1}$ and then recover the background skill grid using $\omega_0^i=\exp\left(\mu_{\omega_0}+y^i\cdot\sigma_{\omega_0}\right)$, with $\omega_0^i\in\left\{\omega_0^1,...,\omega_0^N\right\}$. The PMF of each type is given by $\pi\left(\omega_0^i\right)$ for $i=1,...,N$ and computed as follows: $\Phi\left(y^i+\frac{\Delta_y}{2}\right)-\Phi\left(y^i-\frac{\Delta_y}{2}\right)$. We set $N=250$, $\Phi\left(y^1\right)=0.00001$ and $\Phi\left(y^N\right)=0.99999$. The PMFs are normalized so that $\sum_{i=1}^{N}\pi\left(\omega_0^i\right)=1$.

E.1.3 Calibration

We fix the values of the following parameters: $(\varepsilon,\rho,\eta)=(0.5,1,0.75)$. Then, we choose the parameters $(B,\mu_{\omega_0},\sigma_{\omega_0})$ to match three empirical moments. Units are calibrated so that every dollar-valued object is measured in thousands of dollars per year. Because the model has a lifetime steady-state interpretation, we abstract from population growth and assume that individuals engage in K-12 education for 13 years

and then work and pay labor income taxes for 30 years (between ages 30 and 60), abstracting from transitional dynamics between education, work, and retirement. Then, we normalize $p = 1$ so that $E(\omega_0)$ measures per-taxpayer annual thousands of dollars spent on their own public K-12 investment, which equals $\frac{13}{30}$ times the annual per-pupil expenditure in public K-12.

To match empirical moments, we solve the model under a fixed policy environment that broadly approximates the actual policy with respect to the vectors T and E. To calibrate the fixed tax system, we use the Piketty, Saez, and Zucman (2018) US-DINA microfiles and fit a linear regression of net taxes paid on pre-tax income to back out an empirical linear tax system. More details on this procedure are found in Section E.4.1 below. This exercise yields $T^{fixed}(z) = 0.303 \cdot z - 12.249$, that is, a universal lump-sum transfer of approximately \$12,250 per capita and a linear tax rate of approximately 30%. To calibrate the fixed education vector, we assume that every type ω_0 receives the same education spending at baseline $E^{fixed}(\omega_0)$. From the U.S. Census Bureau, we get an annual average public spending in K-12 per pupil of \$20,668, so we set $E^{fixed}(\omega_0) = \frac{13}{30} \cdot 20.668 = 8.956$, for all ω_0. This number corresponds to the total K-12 public expenditure, inclusive of current and capital spending, reported in Summary Table 14 of the 2024 Public Elementary-Secondary Education Finance Data (accessed in June 2026).

The three moments we match to calibrate $(B, \mu_{\omega_0}, \sigma_{\omega_0})$ are the following. First, we compute two earnings moments to identify μ_{ω_0} and σ_{ω_0}. We compute labor-income distributions (ages 30–60) in the Panel Study of Income Dynamics (PSID) for the five most recent waves (2015, 2017, 2019, 2021, 2023). Restricting to individuals with strictly positive labor income, we compute the (properly weighted) mean and standard deviation of the pooled panel (after properly adjusting for inflation so that all incomes are measured in 2024 dollars). We compute a mean of \$90,165 and a standard deviation of \$116,575.

Second, we compute one education moment to identify B. This moment is computed as follows. From the Jackson and Mackevicius (2024) meta-analysis, we get that the average effect of spending an extra \$1,000 per pupil per year for 4 years is a 0.0316 standard-deviation increase in test scores. These numbers are in 2018 dollars. Then, after adjusting for inflation, and using the fixed education vector from above, we get a treatment of $(4 \cdot 1,000 \cdot (1 + 0.249)) / (13 \cdot 20,668) = 1.86\%$, where 0.249 is the inflation adjustment, and $13 \cdot 20,668$ is total expenditure per pupil in K-12 (20,668 annual expenditure per pupil times 13 years). Then, the Jackson and Mackevicius (2024) number in terms of our calibration numbers implies that a 1.86% increase in education expenditure increases test scores by 0.0316 standard deviations. To then translate this into an effect on earnings, we use Chetty, Friedman, and Rockoff (2014) to conclude that a one-standard-deviation increase in test scores increases earnings at age 28 by 12.66%. This calculation comes from two normalizations since that paper estimates the effect of a one-standard-deviation increase in teacher value-added (VA) on test scores to be 0.13, and the causal impact of one

standard deviation of teacher VA on age-28 earnings to be \$349.84 (relative to an average of \$21,256), so $0.1266 = (349.84/21,256)/0.13$. Combining all the numbers, we conclude that an increase of 1.86% in $E^{fixed}(\omega_0)$ yields an increase of 0.4% in annual earnings ($0.0316 \cdot 12.66\% = 0.4\%$). Then, the moment we use comes from simulating this thought experiment (increasing education by 1.86%) and estimating the effect on earnings. When doing this, we account for the fact that the numbers from Chetty, Friedman, and Rockoff (2014) are computed in a selected sample with average earnings at 28 of the affected children of \$21,256 (in 2010 dollars) and average earnings of their parents of \$40,808 (in 2010 dollars). Given our steady-state approach, we consider the parents' income, adjusted for inflation, which yields \$58,695, and we further adjust this number by 4% so that it represents average earnings at 45 years old (the average age of the parents in Chetty, Friedman, and Rockoff, 2014 is 40, and DQYDJ's calculation using the Census ASEC/IPUMS CPS suggests a life-cycle adjustment of 4% to translate earnings from age 40 to age 45).[44] This yields average earnings for the population of interest of \$61,029 (68% of our calibration mean earnings).

Using these fixed policy parameters and the moments, we proceed as follows. Given $\left(T^{fixed}, E^{fixed}\right)$ and a guess of $(B, \mu_{\omega_0}, \sigma_{\omega_0})$, we: (1) simulate the ω_0 distribution, (2) apply the Ω function to recover ω, and (3) compute optimal earnings for each type. Then, we (1) compute average earnings, (2) compute the standard deviation of earnings, and (3) identify the population whose average earnings equal \$61,029, increase their fixed education policy by 1.86%, and check by how much their earnings change on average. We repeat this process until the earnings moments match the PSID moments and the education-driven earnings increase in the population of interest matches 0.4%.

The outcome of this procedure yields $(B, \mu_{\omega_0}, \sigma_{\omega_0}) = (2.7787, 3.9462, 1.0896)$. Moments are matched exactly. With these values, the endpoints of the grid in the baseline simulation are $\omega_0^1 = 0.5$ and $\omega_0^N = 5,400$, so that the distribution of annual potential earnings ranges from \$500 to \$5,400,000.

Finally, we use the calibration outcome to recover the exogenous revenue requirement R. We set R as the residual in the budget constraint under the fixed tax system: $R = 0.303 \cdot \int_{\omega_0} z(\omega_0) f(\omega_0) d\omega_0 - 12.249 - 8.9561 = 6.1149$, so, on top of the lump-sum transfer and the education spending, we calibrate per-capita spending of \$6,115, which is held fixed across the optimal policy simulations.

E.1.4 Optimal Policy

Given calibrated parameters, we simulate our baseline distribution of ω_0 and proceed with the optimal policy problem. In our optimal policy simulations, we set $\rho = 1$, so that social welfare is logarithmic.

The optimal policy problem consists of a vector of unknowns of size $2 \times N + 1$: (1) N education

[44] See DQYDJ's average-income-by-age tabulations of the Census ASEC/IPUMS CPS at https://dqydj.com/average-median-top-income-by-age-percentiles/ (accessed August 2026).

parameters (one for each type, according to equation E6), (2) N marginal tax rates (one for each type, according to equation E7), and (3) 1 universal lump-sum transfer recovered using the budget constraint.

The routine works as follows. For a guess of the policy parameters, we (1) apply the Ω function to recover ω, (2) compute optimal earnings for each type (which, in turn, allows us to recover their indirect utilities), (3) compute the budget constraint multiplier $\lambda = \sum_{i=1}^{N} \pi\left(\omega_0^i\right) U\left(\omega_0\right)^{-1}$ to compute the welfare weights, and (4) evaluate the $2 \times N + 1$ social planner's first-order conditions. The routine iterates until all FOCs are satisfied. The no predistribution benchmark solves the FOCs of the N marginal tax rates and the universal lump-sum transfer after imposing $E\left(\omega_0\right) = 0$ for all ω_0.

In evaluating the planner's FOC, we need to adjust the marginal tax rate condition to suit the discrete approximation (see, e.g., Mankiw, Weinzierl, and Yagan (2009); this is not needed for the education FOCs, as they are pointwise in both cases). The adjusted MTR conditions are given by

$$\frac{\tau\left(z\left(\omega_0^i\right)\right)}{1-\tau\left(z\left(\omega_0^i\right)\right)} = \frac{1}{\varepsilon} \cdot \frac{\widehat{\Omega}'\left(\omega_0^i\right)}{\omega\left(\omega_0^i\right) \frac{\pi\left(\omega_0^i\right)}{\Delta_{\omega_0}^i}} \cdot \sum_{j=i}^{N} \left(1 - g\left(\omega_0^j\right)\right) \pi\left(\omega_0^j\right), \tag{E8}$$

where $\Delta_{\omega_0}^i$ is the relevant bin width to transform the PDF to a PMF. Note that the bin width is not constant because our grid is not equally spaced in the ω_0-space and because a non-constant education vector further stretches the distribution. Since $\Delta_{\omega}^i = \widehat{\Omega}'\left(\omega_0^i\right) \cdot \Delta_{\omega_0}^i$, we can implement this numerically by computing the numerical gradient of $\widehat{\Omega}\left(\omega_0\right)$ at ω_0^i as $\Delta_{\omega}^i = \frac{1}{2}\left(\omega^{i+1} - \omega^{i-1}\right)$ (with one-sided differences at the endpoints).[45]

Likewise, the discrete version of the budget constraint is given by

$$\sum_{i=1}^{N} T\left(z\left(\omega_0^i\right)\right) \pi\left(\omega_0^i\right) = R + p \sum_{i=1}^{N} E\left(\omega_0^i\right) \pi\left(\omega_0^i\right), \tag{E9}$$

where $T\left(z\left(\omega_0^i\right)\right)$ is computed as follows. Let $\tau_i = \tau\left(z\left(\omega_0\right)^i\right)$ and let T_0 be the intercept (universal lump-sum). We define $T\left(z\left(\omega_0^1\right)\right) = \tau_1 \cdot z\left(\omega_0^1\right) - T_0$, and, for $i > 1$, we compute

$$T\left(z\left(\omega_0^i\right)\right) = T\left(z\left(\omega_0^{i-1}\right)\right) + \tau_i \cdot \left(z\left(\omega_0^i\right) - z\left(\omega_0^{i-1}\right)\right). \tag{E10}$$

E.1.5 Comparative Statics

We conduct numerical comparative statics exercises. In each comparative static exercise, we solve for the optimal policy for different parametric combinations and compute the Gini coefficient of ω_0 (constant

[45] The discretization implies that the two transversality conditions cannot both be enforced simultaneously. The sum evaluated at $i = 1$ yields $\tau\left(z\left(\omega_0^1\right)\right) = 0$ exactly, but $\tau\left(z\left(\omega_0^N\right)\right)$ may be positive as $\pi\left(\omega_0^N\right)$ is not necessarily negligible, especially when considering the log-spacing of the grid. This nuance has zero quantitative bite for the aggregate results.

across iterations), the Gini coefficient of ω (not constant across iterations, as it depends on the optimal education policy), and the Gini coefficient of after-tax income $z - T(z)$ (not constant across iterations, as it depends on both the optimal education policy and the optimal tax system).

Our first comparative static exercise concerns ε. We compute a grid of values for ε ranging from 0.001 to 1. For each value of ε, we solve the optimal policy problem holding the rest of the calibrated parameters fixed at their baseline values. However, we note that, under our utility function, the marginal willingness to pay for increased skill $du/d\omega$ is given by

$$\frac{du}{d\omega} = \frac{A\left(1 - \tau\left(z\left(\omega_0\right)\right)\right)^{1+\varepsilon}}{1+\varepsilon}. \tag{E11}$$

This implies that changing ε changes both the marginal efficiency cost of taxation (which is the comparative-static effect we are interested in) and the marginal welfare benefit of education. To isolate the comparative-static effect of the first force, we solve for the A parameter that, under the new ε, would leave the marginal willingness to pay for skill constant at the initial calibration level (that is, under the fixed tax system $T^{fixed}(z)$ described above), so that each iteration changes both ε and A.[46]

Our second comparative static exercise concerns the education elasticity moment. We proceed as follows. Integrating equation (17) across types, we compute the average elasticity of earnings with respect to education at the calibration values. This gives us an average elasticity of 0.196. Based on this value, we use a grid of this elasticity ranging from 0.1 to 0.3, where for each elasticity value, we compute the B parameter that would give that average elasticity at calibration values, holding μ_{ω_0} and σ_{ω_0} fixed. So, essentially, this is a comparative static exercise with respect to B but expressed in units that are interpretable in elasticity terms.

Our third comparative static exercise concerns heterogeneity in marginal education productivity. To evaluate such heterogeneity, we respecify equation (E3) in terms of the scalar parameter $B(\omega_0) = BC_\xi \exp\{\xi F_0(\omega_0)\}$. We choose $C_\xi = \left(\mathbb{E}\left[\exp\{\xi F_0(\omega_0)\}\right]\right)^{-1}$ so that average productivity is held fixed at its baseline value: $\mathbb{E}\left[B(\omega_0)\right] = B$. The parameter ξ represents the spread in education productivity between the top and bottom types: $\xi = \log B\left(\overline{\omega_0}\right) - \log B\left(\underline{\omega_0}\right)$. We consider a grid of ξ ranging from -0.5 to 1.

Our fourth comparative static exercise concerns the inequality aversion parameter ρ. This comparative static does not consider any recalibration. We approximate the Rawlsian case by considering a long grid and numerically finding the values above which the Gini coefficients under optimal policy converge.

[46] Note that, at calibration, the marginal tax rate is constant for all ω_0, which implies that the baseline value of $\frac{du}{d\omega}$ is a constant, so solving for A given $\frac{du}{d\omega}$ and ε is straightforward.

E.2 Simulations With Preference Heterogeneity

Simulations with preference heterogeneity inherit most of the structure and calibration from the baseline case by reinterpreting the former distribution of ω_0 as the distribution of $A\omega_0$ and then considering different combinations of marginal distributions of A and ω_0 that yield similar distributions of $A\omega_0$ but different relative importance of A and ω_0 in explaining the aggregate variance in $A\omega_0$.

In what follows, we define the notation $\widetilde{\omega} = A\omega_0$. We also make the following assumptions: (1) A and ω_0 are independent, and (2) A and ω_0 are distributed log-normal, with parameters (μ_A, σ_A) and $(\mu_{\omega_0}, \sigma_{\omega_0})$. Then, the distribution of $\widetilde{\omega}$ is also log-normal with parameters $(\mu_{\widetilde{\omega}}, \sigma_{\widetilde{\omega}})$, which we know from the baseline simulation with no preference heterogeneity. That is: $(\mu_{\widetilde{\omega}}, \sigma_{\widetilde{\omega}}) = (3.95, 1.09)$.

We first describe how we analytically characterize the marginal distributions of A and ω_0, and then describe how we simulate them. Finally, we describe the solution of the optimal policy problem.

E.2.1 Characterizing Marginal Distributions

Let A be log-normal with parameters (μ_A, σ_A) and ω_0 be log-normal with parameters $(\mu_{\omega_0}, \sigma_{\omega_0})$. Define $\widetilde{\omega} = A\omega_0$. If A and ω_0 are independent, known properties yield that $\widetilde{\omega}$ is log-normal with parameters $\left(\mu_A + \mu_{\omega_0}, \sqrt{\sigma_A^2 + \sigma_{\omega_0}^2}\right)$. We also know that, if X is log-normal with parameters (μ, σ), then

$$\mathbb{E}[X] = \exp\left(\mu + \frac{\sigma^2}{2}\right), \tag{E12}$$

$$\mathbb{V}[X] = \left(\exp\left(\sigma^2\right) - 1\right) \cdot \exp\left(2\mu + \sigma^2\right). \tag{E13}$$

The goal of the exercise is to find different combinations of $(\mu_A, \sigma_A, \mu_{\omega_0}, \sigma_{\omega_0})$ such that:

- $\mathbb{E}[\widetilde{\omega}]$ and $\mathbb{V}[\widetilde{\omega}]$ are constant across cases (specifically, equal to the values inherited from the case with no preference heterogeneity).
- $\mathbb{E}[A]$ and $\mathbb{E}[\omega_0]$ are constant across cases (also with values known from the baseline case with no preference heterogeneity).
- The variance of $\widetilde{\omega}$ explained by the variance of A is equal to some value. We use the R^2 of a (properly weighted) regression of $\widetilde{\omega}$ on A, because it (1) is between 0 and 1 (with 0 the limit equal to our baseline simulation with no preference heterogeneity and 1 the limit case of only preference heterogeneity), (2) is interpretable in levels and can potentially be estimated empirically, and (3) can be determined analytically as follows. The R^2 of the proposed regression is given by

$$R^2 = \frac{\left(\mathbb{C}\left(\widetilde{\omega}, A\right)\right)^2}{\mathbb{V}\left(\widetilde{\omega}\right)\mathbb{V}\left(A\right)}. \tag{E14}$$

Also, we know that:

$$\mathbb{C}\left(\widetilde{\omega}, A\right) = \mathbb{E}\left[A^2\omega_0\right] - \mathbb{E}\left[A\right]^2\mathbb{E}\left[\omega_0\right] = \mathbb{E}\left[\omega_0\right]\left(\mathbb{E}\left[A^2\right] - \mathbb{E}\left[A\right]^2\right) = \mathbb{E}\left[\omega_0\right]\mathbb{V}\left[A\right], \tag{E15}$$

which implies that:

$$R^2 = \frac{\left(\mathbb{E}\left[\omega_0\right]\mathbb{V}\left[A\right]\right)^2}{\mathbb{V}\left(\widetilde{\omega}\right)\mathbb{V}\left(A\right)} = \frac{\left(\mathbb{E}\left[\omega_0\right]\right)^2\mathbb{V}\left[A\right]}{\mathbb{V}\left(\widetilde{\omega}\right)} = \frac{\exp\left(\sigma_A^2\right) - 1}{\exp\left(\sigma_A^2 + \sigma_{\omega_0}^2\right) - 1}, \tag{E16}$$

where the last equality comes from replacing the moment definitions and simplifying terms.

With this strategy, we have five moments and four parameters. However, some of these equations are redundant, yielding an exactly identified set of equations:

- We know, from the baseline case with no preference heterogeneity, that $\mathbb{E}\left[\widetilde{\omega}\right]$ and $\mathbb{V}\left[\widetilde{\omega}\right]$ determine $\mu_{\widetilde{\omega}} = \mu_A + \mu_{\omega_0}$ and $\sigma_{\widetilde{\omega}}^2 = \sigma_A^2 + \sigma_{\omega_0}^2$ using equations (E12) and (E13). In particular, $(\mu_{\widetilde{\omega}}, \sigma_{\widetilde{\omega}})$ are the log-normal parameters that we calibrate for ω_0 for the case with $\sigma_A = 0$ and $\mu_A = 0$ (which yield $\mathbb{E}\left[\omega_0\right] = \mathbb{E}\left[\widetilde{\omega}\right]$, $\mathbb{V}\left[\omega_0\right] = \mathbb{V}\left[\widetilde{\omega}\right]$, $\mathbb{E}\left[A\right] = 1$, and $\mathbb{V}\left[A\right] = 0$).
- Then, given $\sigma_{\widetilde{\omega}}^2$ and R^2, we can identify σ_A using equation (E16) (as we can write σ_{ω_0} as a function of $\sigma_{\widetilde{\omega}}^2$ and σ_A).
- Then, given σ_A and $\mathbb{E}\left[A\right]$, we can identify μ_A using equation (E12).
- Finally, given (μ_A, σ_A) and $(\mu_{\widetilde{\omega}}, \sigma_{\widetilde{\omega}})$, we can identify $(\mu_{\omega_0}, \sigma_{\omega_0})$ with the identities $\mu_{\widetilde{\omega}} = \mu_A + \mu_{\omega_0}$ and $\sigma_{\widetilde{\omega}}^2 = \sigma_A^2 + \sigma_{\omega_0}^2$.

This procedure provides us with the exact combinations of $(\mu_A, \sigma_A, \mu_{\omega_0}, \sigma_{\omega_0})$ for each R^2 that generate constant distributions of $A\omega_0$ but with controlled relative variances of A and ω_0 to match different values of R^2.

E.2.2 Simulation of Marginal Distributions

The simulations of the log-normal marginal distributions inherit the same procedure as in the baseline case. That is, for each set of parameters $(\mu_A, \sigma_A, \mu_{\omega_0}, \sigma_{\omega_0})$, we generate grids $A^i \in \left\{A^1, ..., A^N\right\}$ and $\omega_0^i \in \left\{\omega_0^1, ..., \omega_0^N\right\}$, where $A^i = \exp\left(\mu_A + y^i \cdot \sigma_A\right)$ and $\omega_0^i = \exp\left(\mu_{\omega_0} + y^i \cdot \sigma_{\omega_0}\right)$, with y^i and its corresponding density as defined in Section E.1.2 above. We consider $N = 250$ for each marginal distribution, matching the grid size of the baseline case. To double-check that our discretization procedure does not compromise the exact analytical result described above, we confirm that the R^2 of the regressions of $A\omega_0$ against A using the discrete grids and their PMFs match the R^2 values with substantial precision.

E.2.3 Optimal Policy

In this case with preference heterogeneity, the solution to the optimal policy problem consists of the education vector of constant size N, a vector of marginal tax rates, and the universal lump-sum transfer. A challenge in this simulation is that earnings in terms of primitives are a function of combinations of A and ω_0 (through the object $A\hat{\Omega}(\omega_0)$) which, in terms of the discrete approximation, implies that the tax schedule should be defined over $N \times N$ points, which is not tractable ($250^2 = 62,500$). In addition to this dimensionality problem, also because of the discrete approximation, the raw PMF of $A\omega_0$ built from the marginal distributions is non-smooth, which generates issues in the optimization of marginal tax rates.

To deal with these challenges, we restrict the profile of marginal tax rates to be defined over a grid of N points (instead of N^2) and proceed as follows:

- For a guess of the policy parameters, we apply the education vector to the marginal distribution of ω_0 to recover the ω vector.
- Then, we recover the raw distribution of $A\omega$ by considering all the unique combinations between the grid points in A and ω, with their corresponding PMFs (given the independence assumption, the PMF of $A\omega(\omega_0)$ is just the PMF of A times the PMF of ω_0).
- Then, we define a grid of $A\omega$ of size N which is log-spaced between the percentiles 0.0001 and 0.9999 of the raw discrete distribution of $A\omega$ (of dimension N^2). These are the N points for which the profile of marginal tax rates is defined. We assign mass to each bin by smoothly grouping the raw bins that correspond to each grid point bin (smoothly means that the mass is distributed between contiguous grid points to avoid discrete jumps in the numerical solver when changes in the education vector reclassify types across grid points).
- Then, for each $A\omega$ combination we recover the marginal tax rate they face by using linear interpolation of the marginal tax rate schedule defined over the coarser grid of size N. Assigning a marginal tax rate for each type is needed for computing the fiscal externalities relevant to the education FOC and the budget constraint.
- Having recovered the marginal tax rates for all $A\omega$ types, we can compute the actual taxes paid by each type, using a similar sequential logic as in the case with no preference heterogeneity.
- To compute each $A\omega$ welfare weight, we compute a hypothetical distribution of $A\omega$ where we replace A with $A^* = \mathbb{E}[A]$ (since $\mathbb{E}[A] = 1$, this hypothetical distribution coincides with the marginal distribution of ω_0). For this hypothetical distribution, we can recover the marginal tax rate by linear interpolation, compute optimal earnings, compute taxes paid by linear interpolation of the

taxes paid schedule on the full $A\omega$ raw grid, and then compute the hypothetical indirect utility. We normalize this indirect utility by the average social marginal utility of this hypothetical distribution to compute the welfare weights. This gives us a vector of ω_0-specific welfare weights.

- Then, we compute the planner's FOCs as follows. For the education FOCs, we recover the average fiscal externality for each type by taking the average across A for a fixed ω_0, using our previously computed matrix of marginal tax rates for all $A\hat{\Omega}(\omega_0)$ combinations. The marginal willingness to pay for increased skill is recovered using the previously computed ω_0-specific welfare weight and computing the hypothetical marginal willingness to pay for skill using the marginal tax rate that applies to each hypothetical case.
- For the marginal tax rate FOCs, we evaluate the FOC in each of the N points of the log-spaced grid of $A\omega$ of size N, considering the average welfare weight within each bin, which is computed numerically given the types that are allocated to the bin.
- Finally, for the budget constraint, we proceed with the exact solution as we know the taxes paid by each $A\omega$ type of the raw distribution.

This procedure is repeated until all FOCs are satisfied. This is conducted separately for each R^2.

E.3 Simulations With Parental Education Investments

The simulations with parental investments abstract from preference heterogeneity but allow for parental investments according to the model presented in Section 3.1, with their optimal decisions made according to their first-order condition specified in equation (20). As detailed in Section 4.5, we assume that the skill production function takes a CES form:

$$\omega = \Omega\left(\omega_0, E_p\left(\omega_0\right), E_g\left(\omega_0\right)\right) = \omega_0 + B\left[E_p\left(\omega_0\right)^{\delta} + E_g\left(\omega_0\right)^{\delta}\right]^{\eta/\delta}. \tag{E17}$$

We recalibrate the parameters $(B, \mu_{\omega_0}, \sigma_{\omega_0})$ using the same procedure as in the baseline case with no preference heterogeneity and inactive parents, adjusting for the fact that parents invest in their children and, therefore, affect the target moments of the income distribution and the education elasticity. In this calibration, we set $\kappa = 1$ and $\delta = \eta = 0.75$. The latter implies that, at baseline, there is neither crowd-in nor crowd-out ($\Omega''_{pg} = 0$), which allows us to have an analytical solution to parental investments:

$$E_p(\omega_0) = \left[\frac{\kappa A(1-\tau(z(\omega_0)))^{1+\varepsilon} B\eta}{p(1+\varepsilon)}\right]^{1/(1-\eta)}. \tag{E18}$$

Because the calibration assumes a constant marginal tax rate, $E_p(\omega_0)$ is the same for all ω_0. This recalibration procedure yields $(B, \mu_{\omega_0}, \sigma_{\omega_0}) = (2.8593, 3.9078, 1.1043)$.

In our comparative statics exercises, we again compute Gini coefficients for background skill ω_0, realized skill ω, and after-tax income $z - T(z)$, to assess the relative importance at the optimum of both predistribution and redistribution. However, we decompose the difference between inequality in ω_0 and ω in two different channels. First, we compute the Gini coefficient for $\Omega(\omega_0, E_p(\omega_0), 0)$, i.e., the children's skill after parental investments but before government intervention. Then, we define predistribution as the difference between inequality in this object and inequality in ω, so we isolate the role of government investments (conditional on parental investments).

E.3.1 Comparative Statics wrt. κ

The first comparative static exercise concerns κ, where we vary it from 0 to 1.5 in 0.1 increments. We set $\delta = \eta = 0.75$ and solve the optimal policy problem for each value of κ. At the optimal policy, $E(\omega_0)$ is no longer constant because marginal tax rates vary across types. Because $\Omega''_{pg} = 0$, the government FOC for education is unchanged (see Proposition 4). However, the government's FOC for marginal tax rates changes for two reasons. First, as shown in Proposition 3, the efficiency cost of taxation comprises two behavioral responses, measured by the sum of ε and ε_p. The first one is the standard earnings response of the children. The second is the parental response in educational investments which, combining equation (24) with equations (E17) evaluated at $\delta = \eta$ and (E18), is given by

$$\varepsilon_p = \frac{B\eta(1+\varepsilon)E(\omega_0)^\eta}{(1-\eta)\omega}. \tag{E19}$$

The second adjustment is that, because ω is now affected by parental responses to taxes, the relevant density for assessing the mechanical effects (that hold behavior constant) needs not include the changes in the parental investments. Said differently, the change of variables needed to go from earnings to types needs to hold behavior constant; if not, the parental response would be counted twice: in the elasticity concepts and in the density computations. Then, the numerical gradient of ω would be double-counting parental responses. Because of that, the numerator of the second term in the marginal tax rate FOC is given by $\Delta^i_{\omega_0} + B\eta E_g^{\eta-1}\Delta^i_{E_g(\omega_0)}$, which only coincides with Δ^i_ω when parents are inactive. Again, we compute $\Delta^i_{\omega_0}$ and $\Delta^i_{E_g(\omega_0)}$ using numerical gradients.

E.3.2 Comparative Statics wrt. ν

The second comparative static exercise concerns ν, where we vary it from 0 to 0.8. In this exercise we also fix $\delta = \eta$. Recall that ν corresponds to the elasticity of parental weight with respect to type, following

the isoelastic formulation $\kappa(\omega_0) = C_\nu \omega_0^\nu$ with C_ν constant, so that $d\log\kappa/d\log\omega_0 = \nu$. This comparative static varies the spread in κ across types, with high-ω_0 types having higher parental weights κ. The strength of the relationship is governed by ν. Because we want to isolate the effect of dispersion in κ but not its average level, we choose C_ν such that, for each ν, $\mathbb{E}[\kappa(\omega_0)] = 1$. It is straightforward to see that the C_ν that ensures that condition is ν-specific and given by $C_\nu = 1/\mathbb{E}[\omega_0^\nu]$. Because ω_0 is distributed log-normal, it follows that $\mathbb{E}[\omega_0^\nu] = \exp\left(\nu\mu_{\omega_0} + \frac{1}{2}\nu^2\sigma^2_{\omega_0}\right)$.

Again, because $\Omega''_{pg} = 0$, the government FOC for education is unchanged (see Proposition 4). However, the government FOC for marginal tax rates changes because of the same two reasons discussed above. The computation of ε_p does not change with heterogeneous κ relative to the homogeneous case discussed above; however, heterogeneous κ plays a role through its effect on $E(\omega_0)$. Also, allowing for heterogeneous κ requires a small correction in the change-of-variable term because now, locally, parental investments change for two reasons: different marginal tax rates (which we don't have to consider in the density term given that it relates to the mechanical effect) and differential altruism (which we do have to consider in the density term as it relates to a change in primitives, so parental investments change even assuming away behavioral responses to taxes). So the numerator of the second term in the marginal tax rate FOC is now given by $\Delta^i_{\omega_0} + B\eta E_g^{\eta-1}\Delta^i_{E_g(\omega_0)} + B\eta E_p^{\eta-1}\frac{\partial E_p}{\partial\kappa}\Delta^i_{\kappa(\omega_0)}$. We can conveniently write $\frac{\partial E_p}{\partial\kappa}\Delta^i_{\kappa(\omega_0)} = \frac{E_p(\omega_0)}{1-\eta}\frac{d\log\kappa}{d\omega_0}$, which implies that the third term reduces to $B\eta E_p^{\eta-1}\frac{\partial E_p}{\partial\kappa}\Delta^i_{\kappa(\omega_0)} = \frac{\nu}{1-\eta}\frac{B\eta E_p^\eta}{\omega_0}$.

E.3.3 Comparative Statics wrt. ϕ

Finally, the third comparative static exercise concerns $\phi = 1/(1-\delta)$: the elasticity of substitution between parental and public education investments. This exercise assumes $\kappa = 1$ for all ω_0. When $\delta < \eta = 0.75$ ($\phi < 4$), we are in a situation with crowd-in ($\Omega''_{pg} > 0$). When $\delta > \eta = 0.75$ ($\phi > 4$), we are in a situation with crowd-out ($\Omega''_{pg} < 0$). We consider a grid of ϕ between 2 and 7, and also consider the limit case of $\delta \to 1$ ($\phi \to \infty$), where private and public investments are perfect substitutes. Importantly, when $\delta \neq \eta$, parental investments no longer have an analytical solution, as they depend non-linearly on public investments (see equation 20). Therefore, this comparative static simultaneously solves for the $3N+1$ unknowns fixed point that consists on N private education investments and the $2N+1$ policy parameters: N public education investments, N marginal tax rates, and the lump-sum transfer.

This comparative static also requires adjusting the planner's FOCs. Regarding public education, it follows from Proposition 4 that the marginal effect of public education is now given by $\Omega_g - \frac{\Omega_{pg}\Omega_p}{\Omega_{pp}}$, so the FOC of public education is adjusted accordingly.[47] Regarding the marginal tax rate FOCs, similar

[47] We note that, when δ is large enough, this term may induce a non-concavity in the planner's problem for some types, where the composite object $\Omega_g - \frac{\Omega_{pg}\Omega_p}{\Omega_{pp}}$ might become non-decreasing in E_g. The optimal policy solvers start displaying this non-concavity problem in a small set of types when ϕ is above 7.5.

adjustments as before are introduced. First, the ε_p correction is no longer independent of $E_g(\omega_0)$ and, therefore, is computed using the general expression of equation (24). Second, the correction to the density term is analogous to the case with homogeneous κ discussed above; however, the dependence between private and public education implies that the term is given by $\Delta^i_{\omega_0} + \left(\Omega_g - \frac{\Omega_{pg}\Omega_p}{\Omega_{pp}}\right)\Delta^i_{E_g(\omega_0)}$.

Finally, to compute the solution at the limit with $\phi \to \infty$, we note that with perfect substitutability, the non-negativity constraint on $E_p(\omega_0)$ binds (this is not true for finite ϕ as the functional forms we assume satisfy Inada conditions and, therefore $E_p(\omega_0) > 0$ for $\phi < \infty$). Concretely, this means that, when $E_g(\omega_0) > 0$, then $E_p(\omega_0) = 0$. From the planner's perspective, the optimality of investing in public education and making parents inframarginal depends on how the optimal total education target relates to the laissez-faire parental investment. Noting that optimal public education investments are progressive, and that parental investments are of smaller magnitudes, this reduces the problem to finding a threshold of ω_0 (say, $\widehat{\omega}_0$) such that, for all $\omega_0 < \widehat{\omega}_0$ the government invests in education and fully crowds out parents, and for all $\omega_0 > \widehat{\omega}_0$ the government sets $E_g(\omega_0) = 0$ and lets parents invest. We numerically implement this by solving the optimal policy problem for all possible thresholds and selecting the one that maximizes social welfare. We note that, for parents at the left of the threshold, the FOCs are equivalent to the baseline case with inactive parents because parents are no longer at the margin of investment. On the other hand, for parents at the right of the threshold, the marginal tax rate FOCs require the adjustments specified above but the education investment is set to $E_g(\omega_0) = 0$. The solution locates $\widehat{\omega}_0$ at the 95th percentile of the type distribution and, because parental investments are small in magnitude, this nuance has very little impact at the aggregate (public investment accounts for 99.9% of education investments).

E.4 Further Details

E.4.1 Computing the Empirical Tax System Using PSZ

To estimate the empirical tax system $T^{fixed}(z)$ used in the calibration procedure described above, we pool the US-DINA microfiles of Piketty, Saez, and Zucman (2018) between 2016 and 2020 and proceed as follows. First, we keep tax units where all members are between 30 and 60 years old and collapse the data to the tax unit level (single or married filing). Then, we restrict to wage earners by only keeping tax units where wage income as a share of total fiscal income is between 95% and 105%. Then, we compute income, taxes paid, and transfers received (excluding education expenditures), and then adjust by inflation so values are measured in 2024 dollars. Finally, we run a regression of net taxes paid (taxes minus transfers) on income, controlling for number of children, marital status, and flexible interactions. The coefficient on income is the linear marginal tax rate (30.3%), and the intercept is the lump-sum transfer ($12,249).

F Gini Coefficient and Constant Education

Consider a vector $\{x_i\}_1^n$. The Gini coefficient is given by:

$$G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n^2 \bar{x}} = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n \sum_{i=1}^n x_i}. \tag{F1}$$

Consider the following application. Our random variable is $x = \widetilde{\omega} = A\omega_0$. Assume the provision of education is constant (e.g., because there is no variation in welfare weights). So $y = A\omega = A\left(\omega_0 + \overline{E}\right)$, where $\overline{E} = BE\left(\omega_0\right)^\eta > 0$ is the constant provision of education ($E\left(\omega_0\right)$ is the same for all ω_0).

Consider two polar cases. First, there is no preference heterogeneity, so A is constant but ω_0 follows some distribution. Then, we have that:

$$\begin{aligned} G_x = \frac{\sum_{i=1}^n \sum_{j=1}^n |A\omega_{0i} - A\omega_{0j}|}{2n \sum_{i=1}^n A\omega_{0i}} &= \frac{\sum_{i=1}^n \sum_{j=1}^n |\omega_{0i} - \omega_{0j}|}{2n \sum_{i=1}^n \omega_{0i}} \\ &= \frac{\sum_{i=1}^n \sum_{j=1}^n \left|\omega_{0i} + \overline{E} - \omega_{0j} - \overline{E}\right|}{2n \sum_{i=1}^n \omega_{0i}} \\ &> \frac{\sum_{i=1}^n \sum_{j=1}^n \left|\omega_{0i} + \overline{E} - \omega_{0j} - E\right|}{2n \sum_{i=1}^n (\omega_{0i} + \overline{E})} \\ &= \frac{\sum_{i=1}^n \sum_{j=1}^n \left|A(\omega_{0i} + \overline{E}) - A(\omega_{0j} + E)\right|}{2n \sum_{i=1}^n A(\omega_{0i} + \overline{E})} = G_y. \end{aligned} \tag{F2}$$

So, when there is dispersion in ω_0 but not in A, a uniform education policy decreases the Gini. This is what we observe in the limit point of Figure A.2.

Now, consider the opposite case where there is full preference heterogeneity, so ω_0 is constant but A follows some distribution. Then, we have that:

$$\begin{aligned} G_x = \frac{\sum_{i=1}^n \sum_{j=1}^n |A_i\omega_0 - A_j\omega_0|}{2n \sum_{i=1}^n A_i\omega_0} &= \frac{\sum_{i=1}^n \sum_{j=1}^n |A_i - A_j|}{2n \sum_{i=1}^n A_i} \\ &= \frac{(\omega_0 + \overline{E}) \sum_{i=1}^n \sum_{j=1}^n |A_i - A_j|}{(\omega_0 + \overline{E}) 2n \sum_{i=1}^n A_i} \\ &= \frac{\sum_{i=1}^n \sum_{j=1}^n \left|A_i(\omega_0 + \overline{E}) - A_j(\omega_0 + \overline{E})\right|}{2n \sum_{i=1}^n A_i(\omega_0 + \overline{E})} = G_y. \end{aligned} \tag{F3}$$

So, when there is dispersion in A but not in ω_0, a uniform education policy does not affect the Gini. This is what we observe in the limit point of Figure 9.